NUMERACY ACADEMY

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NUMBER SENSE

PART 1:
TEACHING CHILDREN MATHS
10 HOURS

ADDITIVE OPERATIONS AND PATTERNS

PART 2:
TEACHING CHILDREN MATHS
10 HOURS

MULTIPLICATIVE OPERATIONS AND FRACTIONS

PART 3:
TEACHING CHILDREN MATHS
10 HOURS

SPACE AND SHAPE, MEASUREMENT AND DATA HANDLING

PART 4:
TEACHING CHILDREN MATHS
10 HOURS

TEACHING CHILDREN MATHS

40 HOURS

Funda Wande
Reading for Meaning

Bala Wande
Calculating with Confidence
Numeracy Academy

A team of writers from Bala Wande developed the Mathematics content of the Numeracy Academy drawing on the Bala Wande Thinking Maths modules, in consultation with Cally Khune of RED INK. The materials also draw on the Bala Wande Foundation Phase materials (Grades R to 3) were developed in consultation with a reference team of early Mathematics specialists.

www.fundawande.org
www.redink.org.za

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Bala Wande Grade R Mathematics Programme Teacher and Teacher Assistant Training Facilitator Guides. (2021, 2022).

Bala Wande Grade R-3 Mathematics Programme Teacher Guides and Learner Activity Books. (2021, 2022, 2023).


Welcome and Orientation

Welcome to the Numeracy Academy - this course aims to get you to start Thinking Maths!
The materials are divided into 4 parts. Each part should take you 10 hours to complete.
It is important that you go through the lessons in sequence as each lesson builds on the content from the previous one.
We encourage you to be an active reader while engaging with each lesson.

Each lesson has **video(s)** that you need to watch by clicking watch now. If you are reading the print version of the booklet you can use the QR code to access the video.

Each lesson also has a **self assessment** that you should complete. This will give you a chance to recap on what you learned in the lesson.

Each lesson ends by providing you an opportunity to **reflect** on what you have learned in the lesson. Take time to do this activity - it will help you consolidate and action the learning in your classroom.

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**Check your understanding: Multiple Choice**

1) Children use play to learn:
   - A: cognitively and communicatively
   - B: socially, emotionally and physically
   - C: Both A and B

2) Co-opted play is:
   - A: initiated and directed by the teacher
   - B: initiated and directed by the child
   - C: free time for the teacher

3) Snakes and Ladders is an example of:
   - A: Physical play
   - B: Symbolic play
   - C: Games with rules

4) Play is:
   - A: Important for the development of an executive function
   - B: Only important for breaks
   - C: Worthwhile including if there is extra time in the curriculum

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**Reflection**

- Reflect on your own school experience as a child. Did you have opportunities to play as a means of learning?
- Think about your own practice. Does your teaching provide opportunities for children to play? Or are your lessons mainly teacher-led? Write a personal goal in relation to enhancing the teaching and learning in your classroom by including playful experiences.
In order to gain the most from this course, please ensure that you watch the videos in full and that you complete each self-assessment. The assessments during the course are self-checks and the answers are given at the end of each lesson. As part of the final assessment of the module there will be two tests.

- Test 1 is taken after the completion of Part 1 and 2.
- Test 2 after the completion of Part 3 and 4.

- Each test lasts 1 hour and is in multiple-choice format.
- An online link for each test will be provided on the scheduled date.
- You will receive your results after clicking the submit button at the end of each test.
- If you fail the test you will be provided a second chance to take the test and a new date will be scheduled for this.

We hope you enjoy the course and find it beneficial!
In Part 1 you will be introduced to a play-based approach to teaching and learning, emergent literacy, smart counting and equal exchange. Children learn through play and interaction with other children and adults, and it is important for teachers to use play to enhance their teaching practices and to improve learning opportunities for children. Through this module you will discover that the different forms of play can help teachers to engage children in the active learning of number concept.
In this lesson, we will look at the benefits of play on children’s development. We will clarify the characteristics of play, considering how the choice and control of play activities influences children’s learning. We will also discover more about the five types of free play, thinking about how these can be applied to our own classroom environments.

**What you will learn in this lesson**

- The benefits and value of play for children’s development
- The principles and characteristics of play
- A play continuum, considering the levels of initiation (choice) and direction (control)
- Five types of free play

**The Benefits and Value of Play**

Play is the most important way that babies, toddlers and young children develop and learn across all developmental domains including physical, social, emotional, communicative and cognitive domains.

They use these experiences to make sense of the world, and repeating these experiences strengthens the connections in their brains so that they become permanent. When children play, they are deeply involved in activities which allow them to develop their executive function skills. Executive function is a set of thinking processes which work together to help people remember information, pay attention during an activity, filter out distractions, control their impulses, think creatively and solve problems, and be flexible to adjusting their plans.
ACTIVITY 1

The Power of Play

Watch the video “The Power of Play – A Learning Tool for a Powerful Future” (UNICEF South Africa, LEGO Foundation & Department of Basic Education) which beautifully illustrates the wide-ranging benefit of play throughout childhood. (5:00 minutes)

• Jot down the benefits of play that are described in the video.
• Think about your own experience of play as a child in relation to the ideas raised in the video.

Commentary

Understanding the value and benefits of play – “the power of play” – as it is described in the video – is useful to motivate teachers to shift their teaching methods to foreground play activity instead of the more traditional approach. Giving learners multiple, varied opportunities to play helps them to discover new knowledge and to build their concepts and skills.

Holistic development across all domains is best facilitated by a rich play-based experience. Teachers can use this understanding of the benefits and importance of play for learning to explain to parents and caregivers why play takes a dominant role in their classroom (as opposed to the “work” that the parents expect to happen at school i.e. writing, reading and doing sums). After all, “play is the work of the child” (Maria Montessori, 1949)

Principles and Characteristics of Play

Play is now recognised as a valuable teaching tool, because it can support and nurture children’s learning in powerful ways, and because it allows children to construct their own understanding. However, play is actually difficult to define because people have different social and cultural contexts which influence their understanding of play. It is for this reason that certain principles or characteristics of play have been determined with regards to play as a means of teaching and learning.

These principles are a useful way for teachers:
• to reflect on the play they see their children engaging in and learning from;
• to create more playful classrooms which support learning, and
• to help assess children’s learning through play.
What is a play continuum?

A continuum is a range or series of things that are slightly different from each other, but they lie somewhere between two different possibilities (merriam-webster.com).

What might a play continuum look like?

The play continuum (adapted from Zosh et al. 2018)

Each form of play is decided by who starts/initiates, chooses, and controls/directs the play activity (Zosh et al. 2017). We use the capitals C (Child/ren) or T (Teacher) when the levels of initiation and direction are high. We use the lowercase c (child/ren) or t (teacher) when the levels of initiation and direction of play are low. Look at the table below.

<table>
<thead>
<tr>
<th>Who initiates play?</th>
<th>Who directs play?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who plans, chooses and starts the play activity? Is it the teacher or the child/children?</td>
<td>Who controls, decides and organises the play activity while it is being played? Is it the teacher or the child/children or both?</td>
</tr>
<tr>
<td>A high level of initiation refers to who started the play by doing all, or most, of the planning and choosing.</td>
<td>A high level of direction refers to who has led the play by doing all, or most, of the controlling and organizing during the play.</td>
</tr>
<tr>
<td>A low level of initiation refers to no, or little, planning and choosing by the player/s.</td>
<td>A low level of direction refers to who has done no, or little, controlling and organizing during the play.</td>
</tr>
</tbody>
</table>
ACTIVITY 2

Watch the video "Play continuum" (7:50 minutes) for examples of children engaging in each form of play described in the play continuum as it moves from free play through to playful instruction.

- Observe each example of the different forms of play and think about whether you have observed/used this form of play in your teaching context.
- If so, how is it similar, and different, from the video clips shown here?
- If not, consider why this play is not present in your setting.
- Think about how you might facilitate and encourage the types of play you have not observed from the learners in your class.

Commentary

The video shows children engaging in free play without the involvement of adults, moving to co-opted play where the teacher engages the learners in discussion about what they are doing or provides additional materials or equipment to extend the play possibilities. Both of these examples of co-opted play happen without changing the control or direction of the child/ren.

Instructional teaching (traditional methods e.g. rote counting and drilling facts) is not included on the play continuum because it does not embody the essential characteristics of play. Unsupervised play is also not on the play continuum as young children need to be supervised for safety even when the adults are not involved in directing the children's play.

The five types of free play are:

<table>
<thead>
<tr>
<th>Physical play</th>
<th>Includes active exercise play, fine motor practice and rough-and-tumble play. Important for gross and fine motor coordination and for building strength and endurance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Play with objects</td>
<td>Explore, investigate, and experiment with different objects to develop thinking skills and to learn to problem-solve.</td>
</tr>
</tbody>
</table>
Symbolic play
Use a toy, object, picture, drawing or other mark-making to represent real-life objects.

Pretence and socio-dramatic play
Involves dressing-up and role-playing. Promotes cognitive and social development. Improves self-regulation (the ability to manage own behaviour and thinking).

Games with rules
Learn to follow rules of games, share and take turns and help one another.

ACTIVITY 3
Types of Play
Watch the video "Types of Play" (9:16 minutes) for examples of children engaging in each type of free play illustrated in the Types of Free Play diagram above.

- Observe each example of the different types of free play described in the current literature and think about whether you have observed this type of play in your teaching context.
- If so, how is it similar, and different, from the video clips shown here?
- If not, consider why this play is not present in your setting.
- Think about how you might facilitate and encourage the types of play you have not observed from the learners in your class.

Commentary
All five types of free play are found in all cultures. It is important to remember however, that play may be influenced by how a family or culture value play, and the extent to which adults play with their children. In South Africa, older children tend to have an important role in young children’s learning of traditional games, where play helps with the development of physical agility, concepts, as well as cultural and social learning. In some contexts, older children are instrumental in adapting games for younger children, whereas in other contexts children are encouraged to go around in fairly stable mixed-age groups called ubungani, within which much learning takes place.
It is important to remember that not all families see play in the same way as we do. We need to respect differences and try to understand them.

### Check your understanding: Multiple Choice

1) **Children use play to learn:**
   - A) Cognitively and communicatively
   - B) Socially, emotionally and physically
   - C) Both A and B

2) **Co-opted play is:**
   - A) Initiated and directed by the teacher.
   - B) Initiated and directed by the child.
   - C) Free time for the teacher.

3) **Snakes and Ladders is an example of:**
   - A) Physical play.
   - B) Symbolic play.
   - C) Games with rules.

4) **Play is:**
   - A) Important for the development of executive function.
   - B) Only important for break time.
   - C) Worthwhile including if there is extra time in the curriculum.

### REFLECTION

- Reflect on your own school experience as a child. Did you have opportunities to play as a means of learning?
- Think about your own practice. Does your teaching provide opportunities for children to play? Or are your lessons mainly teacher-led? Write a personal goal in relation to enhancing the teaching and learning in your classroom by including playful experiences.

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**Well done you have completed Lesson 1.**
Play-based learning is a pedagogical approach to teaching and learning that supports healthy and holistic child development. Children learn best by being actively involved with people and objects, and play is the activity that allows children to construct their own knowledge in a hands-on environment.

The play-based approach to teaching and learning recognises that:

- at times children learn best from free play activities initiated and directed by the child without adult involvement
- at other times children learn best from guided play activities that are directed by the teacher (in small or whole groups).

**What you will learn in this lesson**

- Planning and set up for a PBA to teaching and learning
- Implementing a play-based approach in the Maths classroom

**Play-based Approach**

The play-based approach to teaching and learning cycle (as illustrated below) highlights the five essential steps in implementing this approach.

**Plan for learning opportunities:**
- individual, small group and whole class
- balance between child-initiated and teacher-guided activities.

**Prepare environment and materials to support playful learning and exploration.**
- Outdoors
- Indoors
- Materials

**Include a mixture of child-initiated play and teacher-guided playful instruction.**

**Ask:**
- What worked well?
- What didn’t work well? Why didn’t it work well?
- What do I need to do differently next time?

Use the information to plan further teaching and learning.

**Assess children’s learning to:**
- Determine children’s progress
- Plan for future activities

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**LESSON 2:** **PLAY BASED PEDAGOGY**

60 mins

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ACTIVITY 1
Answer the following questions:
• Do you recognize any of these features in your own teaching practice?
• How might you incorporate others that are new into your classroom activities?

Commentary
Many people may assume that play-based learning activities require less planning and preparation than traditional lessons, thinking that children can simply entertain themselves in self-directed activities. It is necessary for you to plan your play-based environment and tasks as carefully as you would any other lesson in order to maximise the learning opportunities. In addition, the assessment and reflection on these teaching and learning opportunities are of equal importance as the planning and set up. You need to use the feedback from the children, and your observations to identify what worked well and what may need to be done differently next time. This will also help you to continue the cycle, and to plan for the next activity by building on the children's previous learning. This will enable the children to deepen and consolidate their understanding.

Setting up for play-based teaching and learning activities.
It is very important for teachers to prepare an interesting, stimulating and playful learning environment. Teachers need to carefully consider where the learning will take place, and how the environment and materials will promote and encourage active engagement. In addition to this, teachers' own attitudes about play will have an important influence on the play-based environment they set up. What the teacher thinks about the idea of play-based learning, how they see the learners, and how they see themselves as the facilitator of play-based learning will affect all the ways in which they go about preparing the learning environment. A teacher who sees the learner as someone who passively receives new information will set up a learning space that is limiting in many ways. On the other hand, a teacher who believes that children actively construct their own understanding through a play-based approach will set up a very different kind of classroom. The environment is a direct mirror of the teacher's thinking about teaching and learning, and about their plans for a play-based approach.

Outdoor Environment
• Play with outdoor materials such as sand, water, ropes, balls, boxes, sticks.
• Use indoor toys (such as construction materials) outside.
• Outdoor activities can inform indoor activities eg: balancing on a plank may inform an indoor construction activity with blocks.

Indoor Environment
• Light, well organised, comfortable, have things in it that are recognisable to the learners.
• Room for learners to move around
• Noisy vs. quiet areas are good to arrange so that learners can play in different ways.
• Flexible design that can be changed as needed.
• Different learning spaces are useful for large group, small group and individual play opportunities

Materials
• Specifically chosen to encourage and support a wide range of play.
• Include unusual materials to encourage experimentation.
• Accessible for all children
• Encourage engagement, challenge and stimulate thinking.
• Allow for creative, open-ended and flexible play opportunities.
• Easily handled by the children, safe, durable and easy to clean and store.
• Manage the number of materials in the environment at any one time.
• Ensure there are enough materials to keep all children productively occupied.
• Rotate materials into the learning environment to expand learners’ experiences.
ACTIVITY 2

Answer the following questions:
• How could you use both indoor and outdoor space to set up your own teaching and learning space?
• How could you use recycled / re-used materials to set up and equip your own teaching and learning space?

Commentary

It is necessary for children to have space to move around during play-based activities, so try to make use of any available space. Learning is not confined to a desk, and children can learn just as effectively when they are playing outside. Just remember to plan and prepare your learning environments carefully, so that there is adequate supervision and resources at all times.

Children do not need bright, shiny and new resources in order to enjoy learning. Children simply need materials and resources that they can physically use themselves as they become active participants in the learning process.

The Intentional teacher

Intentional teachers know that everything they do (or don’t do) impacts on a learner’s development. An intentional teacher plans carefully and acts purposefully so that their learners have optimal learning experiences. Teachers who are intentional are able to explain exactly what they are doing and why they are doing it. The intentional teacher understands children’s development and learning, acts with curriculum goals in mind and uses a range of teaching strategies to mediate learning and development (Epstein, 2007).

An intentional teacher's planning agenda

Inform your planning by:
• observing the learners
• seeing what interests learners
• looking at curriculum requirements
• considering the learning context.

Encourage agency by:
• making the learners feel valued
• giving learners choices
• letting the learners share in decision-making.

Match curriculum themes and topics with the children's interests and the learning context.

Create learning activities that:
• are fun
• are meaningful to learners
• involve social interaction
• are iterative (learners repeat the activities in different ways)
• deeply and actively involve the learners in their own learning.
ACTIVITY 3

Play-based Pedagogy - Maths in the Classroom

Watch the video, “Play-based Pedagogy - Maths in the Classroom” (1:29 minutes) of examples of different types of play activities and games to include in a play-based Maths classroom.

Write a planned play-based activity showing the age group and the links with the curriculum. The planning should:

- link to the curriculum;
- take into account the learners' interests and the context;
- make the learners feel valued;
- offer choice and an opportunity to make decisions;
- be fun;
- be meaningful for learners;
- involve the learners actively in their own learning;
- be iterative (repeat what they are learning in different ways);
- be sociable (engage learners in groups, and in pairs, with another learner or the teacher).

Commentary

In your planned activity, make sure that you have carefully scaffolded the children's learning through active engagement with the environment and the resources. In your planning, check that you have asked questions to get children thinking for themselves, and that there are opportunities for children to talk to each other and to you. Consider whether your activity allows you to model skills, whilst still following the children's lead in terms of allowing them the space to solve problems in their own way.

Intentional teachers observe and assess children as they play and get to know their individual strengths and abilities, and they then plan learning activities that are at the appropriate level. These teachers think about how best to provide children with just the right level of guidance.
Check your understanding: True or False?

1) The five steps involved in the play-based approach teaching and learning cycle are plan, teach, assess, reflect, redirect.

2) The outdoor environment is a worthwhile space to use for play-based activities.

3) Intentional teachers direct children’s learning, and children need to follow the teacher’s lead.

4) Play-based activities in the Maths classroom should be fun and sociable.

REFLECTION

Think about your own Maths lessons.

• Do they contain elements of choice for the children?
• Do they allow children to be deeply involved in the decision-making and problem solving process?
• Are they fun?

Well done you have completed Lesson 2.
Emergent mathematics is the earliest phase of development of Maths concepts, and it incorporates the skills and attitudes that a child develops in relation to mathematical concepts. Children’s understanding of early Maths concepts develops gradually and takes place within real-life contexts and with physical objects.

What you will learn in this lesson

- The early beginnings of number are developed through play.
- Numbers are abstract concepts that are represented in different ways.
- There are different kinds of numbers and number contexts

Early Beginnings of Number

The development of emergent number concepts is similar to the progress shown by children when dealing with printed text in the early stages of reading and writing. In the same way that they make their first attempts to read and show their first writing as scribbles on paper, so do they also begin to understand numbers. Children begin to develop mathematical language and concepts as they measure, construct, compare collections of items, and make patterns in playful activities.
ACTIVITY 1
Think about your own experience at school as a learner, and more recently, as a teacher in relation to early Maths learning.

- What are your earliest memories of learning about number?
- Were your experiences with early Maths all at school or can you remember using number in your home environment?
- How is your teaching practice about number the same/different from your memories as a child?

Commentary
Early years’ teachers should actively introduce mathematical discussions, concepts, and language through a variety of appropriate play-based experiences. Teachers should look for many opportunities to guide children in seeing mathematical ideas and to make connections between these ideas. These opportunities should occur not only during Mathematics focus time, but also across all subjects and during free-play inside and outside the classroom throughout the day. Teachers must encourage children to communicate, and to explain their thinking as they interact with important mathematical ideas in deep and sustained ways.

What is a number?
Numbers are abstract ideas, which we can’t see as we can see objects, such as a table, or a toy. We are not able to hand someone a ‘six’ or a ‘four’ as we could a cup. We use numbers to describe the quantity in a set or collection (e.g., 2 dogs, 4 cups, 1 chair), but because we can’t see numbers, we need to find ways to represent them.

Learners begin to represent numbers by using their fingers, and then gradually start to understand the concept of a number by counting the number of things in a collection and representing the number in different ways, e.g., the object in the collection, a number name (a word), a number symbol (or numeral) or a picture. We refer to this as ‘multiple representations’.
ACTIVITY 2

Write down as many ways as you can think of to introduce the number 6 to a class of Grade R learners.

• Think about how you could represent the number using objects, symbols and actions in a play-based, hands-on activity.

Commentary

Teachers should find many creative, fun ways of introducing numbers to learners. You could use songs, rhymes and stories that focus on the quantity and different representations of the number, for example a story about six animals that live in a house at house number ‘6’ in the road (symbol/numeral). Learners can collect pictures and objects to match the number ‘6’ symbol and the number ‘six’ word. You could then build on the knowledge and interest that learners bring to school and link these stories to familiar contexts that learners can relate to. Other resources can be used, including dot cards, number friezes, counters, beads, and so on.

Different Kinds of Numbers and Contexts

We know that numbers are not concrete objects and that we need to represent them in different ways in order to understand the concept of how much they represent. As learners engage with numbers in their everyday surroundings, they begin to realise that numbers can be used differently in different situations. It is through these encounters with numbers that learners understand that there are different kinds of numbers and that these are used in different ways, for example the number ‘five’ can be:

• a magnitude (a cardinal number): I have five sweets
• a position number (ordinal number): she is fifth in the row
• a label (nominal number): Granny lives at number 5 View Street

Gradually learners begin to understand that the five that expresses that there are five of something, has a different meaning to the five that indicates a house address and the five as in ‘being fifth in a row’.

ACTIVITY 3

Think about how you can help children to see that there are different kinds of numbers.

• Can you think of play opportunities where children mimic a real-life context that develops mathematical ideas?
• Write down one game that children could play that includes mathematical concepts.
• What questions could you ask children during the game to further develop their understanding of the mathematical concepts?
• What mathematical vocabulary needs to be developed through this game to assist the development of the maths concepts?

Commentary

It is important to weave maths into children’s everyday games and activities. In this way, children will realise the relevance of maths, and begin to develop a contextual understanding of number. Teachers can set tasks that covered a wide range of skills, and that will develop children’s understanding of number through play. An example of this is where children pretend to set up a shop. Even with a limited understanding of number, or money, they begin to explore concepts of
‘more than’, ‘less than’ and ‘the same as’. They can even develop notions of cheap and expensive as they bargain and barter for goods. When children work on a project or an investigation, they come across a variety of mathematical problems and questions. Solving these problems helps children to understand mathematical concepts in context, and to make sense of real-life problems. They also become more independent, flexible and able to verbalise their thoughts and ideas.

Check your understanding: Multiple Choice

1) Emergent mathematical understanding develops from:
   A) Working carefully through written problems.
   B) Listening to the teacher’s verbal explanations.
   C) Active participation in play-based experiences.

2) Multiple representations of number:
   A) Help children to make connections between ideas and concepts.
   B) Give children a chance to practice their colouring in skills.
   C) Distracts children from the main focus of the lesson.

3) Multiple representations of number include:
   A) Pictures, symbols, real-life contexts, tally marks, concrete apparatus.
   B) Verbal representations, written representations, pictorial representations, physical representations and real-life problem solving.
   C) Physical resources, drawings, verbal communication, symbolic representations and doodling.

4) Playful opportunities help children to develop:
   A) A conceptual and contextual understanding of maths.
   B) Independence and flexibility.
   C) Both A and B.

REFLECTION

Take some time to think about this session and the importance of early foundations of number development.

• Reflect on your own teaching in terms of the multiple representations of number and playful opportunities for mathematics in context.
• Are these already present in your teaching practice?
• If not, what could you do to include to incorporate them into your daily activities?

Well done you have completed Lesson 3.
LESSON 4: EMERGENT NUMBER (2)

Many early grade teachers are delighted when their learners are able to count by rote and repeat a long string of numbers from 1 to 20 and even beyond. The problem with this is that many learners do not really understand the meaning behind what they are doing. When learners explore numbers and visualise them in many different contexts, they gradually develop a good intuition about numbers and flexible thinking about numbers. We call this their number sense. This lesson introduces you to the kinds of relationships and connections learners should be making with small whole numbers as a foundation to their development of number before they begin to use numbers in operations, such as addition and subtraction, and place value in later grades.

What you will learn in this lesson

• What is subitising?
• Perceptual subitising
• Conceptual subitising

What is subitising?

Subitising is the ability to recognize the numerosity (how many) of a small collection of items, quickly, without counting. It is a fundamental skill in the development of number sense because:

• children begin to know what numbers mean or how many ‘things’ a number refers to;
• children develop pattern recognition; and
• children learn to not over-rely on counting.

Another important aspect of subitising is that it helps children to see how numbers are made up. For example, they could see that the number six can be made up of 3 + 3, or 4 + 2, or 5 + 1. This process lays the foundation for learning how to add and subtract.

ACTIVITY 1

What activities could you use in your classroom to help young children develop their subitising skills?

• Think about how to create opportunities for children to play.
• Include opportunities to use pictures / symbols and concrete resources.

Commentary

You can hold up fingers (on either one hand or two hands) and let children say how many fingers they see. You could also use flashcards with dot patterns or tally marks, and ask the children to tell you how many there are without counting. You can increase the challenge by only showing the cards or your fingers for a few seconds, and seeing if the children can subitise rather than counting. You could also use coloured counters or buttons. If you throw these down and ask children to tell you what they see, they can verbalise how the number is made up. For example, “I see one red counter and three yellow counters. There are four counters altogether”.

60 mins
Perceptual Subitising

Perceptual subitising is the ability to perceive the number of a small collection instantly. This means that learners can recognise how many objects there are immediately without counting them. This type of subitising is only possible with a small number of objects, usually up to five. Being able to recognise four fingers as 4, without needing to count 1, 2, 3, 4, is an important step in the development of a sound number sense. When creating perceptual subitising activities, the arrangement of the dots or counters can be varied so that children can further develop their ability to recognise the represented quantity.

ACTIVITY 2

Find a pair of dice and a pack of playing cards.
• First throw one die and say the number of dots you see as quickly as you can.
• Next throw two dice and say the combined number of dots from both dice as quickly as you can.
• Lastly place the cards face down in their pack. Take one card at a time and say (as quickly as you can) the number items displayed on the card. Don’t read the number symbol! Place the card face down next to the pile and take another card. Repeat for all the cards. You could play this game with the learners in your class or at home with family members.
• How quickly did you manage to say the number of dots/items displayed up to 5? And beyond 5?

Commentary

Subitising small collections of items helps children to understand number words and their association with ‘how many’ (the cardinal value), without having to physically count each item as they say the number names in order. It is a good idea to provide a variety of cards showing numbers represented by different pattern arrangements. The smaller the number of dots on the card the easier it is for learners to subitise the number. This will give them the opportunity to increase their recognition of the visual representations of numbers which will help them as they begin to subitise conceptually.

Conceptual Subitising

The recognition of visual arrangements in perceptual subitising leads to an ability to subitise larger collections. This is called ‘conceptual subitising’, and it is where children are able to make use of number images such as two groups of up to five or arrangements on dice or dominoes. Conceptual subitising enables learners to identify numbers larger than five. This more advanced ability to quickly group and quantify sets helps children to develop their number sense and arithmetic abilities.
For example, learners can immediately recognise that the cards shown below each display seven objects. They can see one group of 5 and one group of 2, and they can put these together to make 7.

**ACTIVITY 3**

**Subitising: How many is it?**

Watch “Subitising: How many is it?”, of a teacher presenting subitising in her classroom. (6:03 minutes)

Think about the ways that she uses dot cards for:
- one-to-one correspondence
- perceptual subitising
- conceptual subitising

**Commentary**

Subitising introduces the basic ideas of cardinality – ‘how many’, ‘more and few/less than’ and ideas of parts and wholes and their relationships. This is extremely important in laying the foundation for a sound understanding of number and for solving problems using the four operations. Conceptual subitising helps children to develop an understanding of part-part-whole relationships, recognising that 3 is part of 4; 2 is part of 5.

How the dots are spatially arranged on the dot cards influences how difficult they are to subitise. It is easiest for all of us to subitise rectangular groupings, followed by linear, circular, and then random arrangements. It does seem as though young children are not able to subitise conceptually and will revert to counting the dots on a card one by one. It is important that teachers introduce dot cards and subitising gradually and do not ‘force’ learners to ‘count’ or put pressure on them to ‘say how many’. To make subitising activities fun, teachers can include auditory activities, such as recognising the number claps or drum beats and linking it with movement and music.
Check your understanding: True or False?

1) Subitising is an important skill because it helps to develop children’s counting ability.

2) Perceptual subitising involves collections of items up to five.

3) Conceptual subitising develops an understanding of part-part-whole relationships.

4) In order to develop subitising skills, teachers should encourage children to verbalise how many dots they see on a card.

REFLECTION

• Think about your teaching and whether you have included subitising as part of your learners’ number development.

• If so, write down as many activities and situations as you can remember.

• If not, consider how you can include this important aspect of number into your teaching and learning and write a list of activities you will include in future teaching.

Well done you have completed Lesson 4.

Answers

1) False - Subitising is an important skill because children learn to not over-rely on counting.
2) True
3) True
4) False - Encouraging children to verbalise how many dots they see tends to make them count rather than subitise as they feel pressurised to come up with the correct answer.
Children’s understanding of number is rooted in counting. Oral counting is the first form of counting. An important step in the development of number is the correct reciting of the names of the numbers in the order of the counting sequence. Initially oral counting (or rote counting) starts from one, i.e., one, two, three...... Later learners become skilled at continuing the counting sequence, starting from other numbers, e.g. five, six, seven, ......

What you will learn in this lesson
- Oral or rote counting
- Jumping tracks and number lines
- Ordinal numbers

Oral or Rote Counting

When young children recite the names of the numbers in the order of a counting sequence it is not always related to the magnitude or numerosity ('how much' or cardinal value) of the numbers. It is learnt as a memorised sequence of words, like a rhyme or song, or as if it is just one long word, e.g. ‘onetwothreefourfive’. In most cases, the counting sequence is learnt and used in the context of daily games and activities and not in the actual process of counting. This is evident, for example, when learners play hide-and-seek, and recite ‘One, two, three ... here I come, ready or not!’ Young children will imitate and extend this ‘verse’ on their own and make up their own counting sequences, such as, ‘one, three, seven, four, ten!’ They gradually learn to recite the sequence of counting words correctly.

Rote or oral counting means reciting aloud the number names in the correct counting order. We refer to these counting numbers as natural numbers, starting from 1, 2, 3, 4 and so on. If we include zero (0) in the set we refer to the numbers as whole numbers.

ACTIVITY 1

Think about the songs and rhymes you learned as a child at home and at school in your home language.
- Use this as an example to explain how songs and rhymes can facilitate the memorisation of the number name counting sequence in that language.
- Do you use songs and rhymes in your Maths lessons?
  - If so, how do you use these?
  - If not, how could you incorporate these into your teaching practice?

Commentary

In the beginning, number names are the same as any new words with no meaning. To remember a sequence of words that don’t have meaning is a difficult task. Imagine that you had to remember a sequence of words and repeat them in order as the sequence gets longer and longer, e.g., chair, hat, box, table, tree, ball, dog, spoon, and so on. In order to recite the counting sequence correctly, learners need many opportunities to practise oral counting.
Learning to count and to produce the correct sequence of number words in English is not a simple process (Gould, 2017). Initially children produce an oral count as a string of number words, later they learn to continue the count from any point, identifying the number word that comes before or immediately follows any counting word.

Children need regular repetition of the counting sequence through rhymes, songs and oral counting in action games to consolidate their learning of the complex skill.

**Jumping Tracks and Number Lines**

Jumping tracks and number lines are used to represent numbers in a continuous way and can be used to reinforce oral counting and the sequencing of the counting words. They also visually indicate that the sequence of numbers are separate entities rather than a long continuous string of words, e.g. 1, 2, 3, 4, 5, ... rather than 12345.... These representations provide a meaningful reference to support the conceptual development of the number word sequence.

Teachers can use number lines as a tool to order numbers and to talk about the relationship between the numbers, e.g. which number come before 3, after 5, between 4 and 6, and so on.

![Number Line](image)

Teachers can draw number tracks so that learners can physically jump backwards and forwards to move within the number sequence. They can begin at any given number and jump on or back from that number and so build their flexibility and fluency with the counting sequence.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

**ACTIVITY 2**

**Sequencing numbers: Oral counting and ordinality**

Watch the video “Sequencing numbers: Oral counting and ordinality” (5.39 minutes), about learning the number names in the correct counting sequence.

- Observe how the teacher uses rhymes, number tracks and number lines to support their memorisation of the number names in order.
- How do the activities in the video help to consolidate learning the number words correct sequence?
- How does the teacher use questions to prompt learners and guide them to respond to her instructions?
Commentary

Learning number names and repeating them in the correct sequence does not necessarily mean that children can count or that they have an understanding of the value of the numbers, but it is an important first step in learning counting skills. Initially children start learning the sequence of number words (one, two, three, etc.) and often confuse the sequence of number words but soon learn to recite the number words in the correct order. In this video the teacher has provided a range of opportunities for learners to count out loud in fun, interactive ways.

Ordinal Numbers

Ordinal numbers describe the position of a person or an object in a line or a list. When we talk about our position in a row we don’t say that we are standing ‘one’ or ‘three’, we use ordinal numbers to say ‘first’ or ‘third’.

<table>
<thead>
<tr>
<th>1&lt;sup&gt;st&lt;/sup&gt;</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt;</th>
<th>4&lt;sup&gt;th&lt;/sup&gt;</th>
<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>6&lt;sup&gt;th&lt;/sup&gt;</th>
<th>7&lt;sup&gt;th&lt;/sup&gt;</th>
<th>8&lt;sup&gt;th&lt;/sup&gt;</th>
<th>9&lt;sup&gt;th&lt;/sup&gt;</th>
<th>10&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>second</td>
<td>third</td>
<td>fourth</td>
<td>fifth</td>
<td>sixth</td>
<td>seventh</td>
<td>eighth</td>
<td>ninth</td>
<td>tenth</td>
</tr>
</tbody>
</table>

The picture below shows children running a race. We can use ordinal numbers to define the position of each runner.

![Children running a race with ordinal numbers](image)  

Teachers need to be aware of the direction that the learners or objects are facing when talking about ‘position’ as the position is related to the direction the learner or object is facing. The person or object in the front of the line is always 1<sup>st</sup> (first) if one is counting from the front.

ACTIVITY 3

- How do you include ordinal numbers in your day-to-day teaching and learning?
- Write a list of games that children could play where they will be able to use ordinal numbers in context as they play.

Commentary

It is important for teachers to help children develop the language of counting and ordinal numbers. In order to do this, they should model the correct use of the new vocabulary by asking questions such as: “Who is in the front? Who is first? Who is second? Who is next? Who is last?”. Teachers should also encourage children to use the new language themselves by asking questions like “What can you tell me about Nosipho’s position?”. Teachers can give children opportunities to play Follow My Leader, running races, racing toy cars or rolling marbles. All of these playful activities will allow children to develop the language of ordinal numbers.
Check your understanding: Multiple Choice

1) Counting out loud is a good indication of:
   A) A good understanding of number.
   B) An ability to remember words in a sequence.
   C) Worthwhile participation in the lesson.

2) Jumping tracks and number lines:
   A) Are only used if children are struggling with rote counting.
   B) Can have numbers arranged in a jumbled order.
   C) Are used to help children see numbers as separate entities.

3) When using ordinal numbers, ‘first’ is:
   A) related to the direction of the people or objects.
   B) Always on the left.
   C) Always on the right.

4) When new mathematical vocabulary is introduced:
   A) Teachers should model correct use of the vocabulary.
   B) Children should use the vocabulary themselves as they participate in the lesson.
   C) Both A and B.

REFLECTION

Think about how you learned to count.

• Was this a fun experience?
• How do you support learners in your class to memorise the number sequences in different number ranges?

Well done you have completed Lesson 5.
Counting is the earliest way a child begins to construct the concept of number, and it is a complex and very important process. Counting is the basis of number concept, and so children need to develop this skill in order to move beyond the rote counting of saying numbers names in a sequence. We discussed rote counting in the previous lesson, so now we will move on to smart counting (or rational counting) which involves counting with understanding.

What you will learn in this lesson

- The knowledge and skills involved in moving from rote counting to smart counting.
- How children begin to understand more about number relationships
- How children progress through different counting strategies as they learn more about number.

Learning to count

As children progress from rote counting to rational counting, they develop specific knowledge and skills along the way.

The six principles shown above clearly identify the knowledge and skills that children learn as they begin to develop their concept of number. Children need to be able to match objects one to one and to verbalise which group has "more" or "less" before they can begin to grasp the 'how manyness' of a number. Teachers need to provide a variety of activities to help children progress from rote counting numbers in a sequence to the understanding that the last number name used when counting indicated the total number in the group.
ACTIVITY 1
Think about the progression of learning children will follow as they move from rote counting to rational counting.

• What kinds of activities should children do as they start to learn to count?
• How can these activities be changed to extend their understanding as they move towards rational counting?

Commentary
The first counting out activities that our learners should do would be counting in ones. Counting in larger groups (such as twos, threes, fives or tens) should be reserved for later when they can count out correctly to at least 100. Counting in larger groups too soon can lead to problems in the understanding of addition at a later stage. Learners are ready to solve addition and subtraction problems when they can count out in ones correctly.

The association between numbers, number names and numerals also needs to be established. The idea of “five-ness” is established by counting five of many different items, in different situations. The name “five” for this number of items is thus established, and the numeral 5 is learnt as the symbol for that number of items.

Once basic counting from one to nine is established, we move on to the need for an understanding of place value to write the numerals for the numbers we are talking about.

Number relationships
As children move from rote counting to rational counting, they begin to expand their understanding of number relationships. Typically, children develop this understanding in three main areas, namely:

• Recognition of patterned sets
• Part-part-whole activities
• Relations with other number

These areas are not always distinct from each other, but for our purposes we can use them to create activities that help children extend their learning. As children learn more about numbers, they begin to see that they are interrelated and connected. The more that this happens, the more meaningful their number sense becomes.

<table>
<thead>
<tr>
<th>Recognition of patterned sets</th>
<th>Part-part-whole</th>
<th>Relations with other numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subitising</td>
<td>Verbalise the grouping and re-grouping of a specific quantity</td>
<td>Consider how numbers are related to other numbers</td>
</tr>
<tr>
<td>Dot cards, Dice, Dominos</td>
<td>Counters / Beans</td>
<td>Ten frame 5, 10, 20 are anchor numbers</td>
</tr>
</tbody>
</table>
ACTIVITY 2

Describe one activity for each of the three areas discussed above.

- What resources will be used?
- Think about how the maths language will be developed (questions, modelling, verbalising).
- What will children learn from each activity?

Commentary

To develop a recognition of patterned sets, you can use dot cards as we discussed in our lesson on subitising. Think about how using two colours of dots could extend children's recognition of number relationships. An understanding of part-part-whole can be developed by using a number mat where a specific number of beans or counters are grouped and re-grouped on either side of the dividing line. Encourage children to use the maths language as they verbalise what they notice about the beans or counters. Ten frames are a great way to help children develop an understanding of number relationships. Children can describe numbers in relation to 5, 10 and 20. For example, they might say “I have 3. I need 2 more to make 5,” or “7 is 3 less than 10.” You could also use flard cards to help children further develop their number sense and their understanding of place value.

Counting strategies

The first counting strategy that learners use is called **counting all**. In this strategy, children identify the quantity of a collection by counting all the objects, each time starting from one. The next strategy used is called **counting on**. Here, children are able to hold one number (usually the first number) in their head and then count on from that number. For example, if a child has 5 in one group and 3 in another, he would start with 5 and count on 6, 7, up to 8. Children then progress further to the next strategy which is **accelerated counting on**. In this strategy, children count on in jumps (of 2, 3, 5, 10 or possibly others) depending on the size of the collection.

ACTIVITY 3

**Smart counting Part 1 (What is it?)**

Watch the video, “Smart counting Part 1 (What is it?)” (4.20 minutes), of a teacher getting her learners to count different representations of the same number shown on the board.

- Have you noticed how the children in your class count?
- Try a similar activity to the one shown in the video in your own class, and observe how the children count.
- What can you do to move children on from counting in ones?
Commentary

Even though a young child might be able to count orally, they may not yet have attached meaning to their counting. Teachers need to observe learners and listen as they count in order to determine whether they have acquired the concept and skills to count collections accurately. Sometimes children count in ones because they have developed it as a habit, and they don’t realise that there is a better way of doing it. Teachers need to model different counting strategies, as well as asking different children to demonstrate their strategies. This helps children to see different ways of counting, and to begin to realise that there may be faster and more accurate ways than their own. Remember to ask questions like “What did you notice about the way Phindi counted?” to help children think about speed and accuracy.

Check your understanding: True or false?

1) Another name for rote counting is smart counting.
2) Children instinctively understand the ‘how manyness’ of a number.
3) Subitising and part-part-whole activities help children to develop an understanding of number relationships.
4) Accelerated counting is the most advanced counting strategy.

REFLECTION

• Think about whether all aspects of number discussed in this session have been covered in the video.
• If not, write down what you think is missing.
• Suggest activities that you think the teacher could have included in her lessons.

Well done you have completed Lesson 6.
In this lesson, we will continue thinking about smart counting. We will focus on how children's extended understanding of number concept helps them to break down and build up numbers, and how this leads them to develop their knowledge of place value.

What you will learn in this lesson
- Breaking down and building up of numbers.
- How children develop an understanding of place value.
- The importance of recognising and addressing children's errors.

Breaking down and building up of numbers
As we learnt in our lessons on play, children learn best through actively participating in playful opportunities. Therefore, as we help children to develop their understanding of number and number relationships, it is a good idea to include activities that allow children to learn through playing games.

Game 123! Who has more? (2 hands)

In this game, children are able to represent a cardinal number using their fingers, using the correct maths language to describe the number. Children are also able to recognise number relationships as they break numbers into two groups, for example 6 is made up of 5 and 1. Games such as these help children to practise conceptual subitising, as well as to prepare them for the way we break down numbers into tens and ones in place value. As children's number concept develops, they begin to understand that numbers can be broken down in different ways, and they start using smart counting to help them identify numbers quickly.
ACTIVITY 1

Smart counting Part 2 (Place value)

Watch the video, "Smart counting Part 2 (Place value)", (3.29 minutes) showing a class practicing smart counting with money.

- What skill is the teacher getting the children to practice?
- How many ways did the children come up with to make R120?
- Can you think of any more ways to make R120?
- Why is this activity important?

Commentary

Children’s basic number development appears to proceed slowly, especially when compared with the relative speed at which children learn language. The ability to recognise, describe and represent numbers and their relationships is one aspect of understanding number. Another is the awareness of the different ways and different contexts in which numbers are used in everyday life. In this activity, children were breaking down and building up numbers, using monetary amounts to help them with their smart counting.

Encourage children to participate in activities involving speedy representations of numbers with fingers or bead strings, and with games like ‘Shake Shake Break’ where learners shake a number of counters in their two hands and ‘break’ them apart to form two groups. These kinds of activities are the first experiences learners have in building up and breaking down numbers. You can use concrete, tangible objects in your activities (such as blocks, toys, fruit etc). You can also use jumps, claps, drum beats, or even more abstract things, such as the days of the week.

Developing an understanding of place value

As children learn about number, the way that they see numbers starts to change. Initially, if a child was shown the number 36, he would see it as a single numeral. At this stage, the child is likely to know the number names (‘thirty’ and ‘six’) but he may still represent the number as 3 blocks (to show the 3 of 36) and 6 blocks (to show the 6 of 36).

As his number concept develops, and he understands the ‘how manyness’ of the number, he may begin to transition to the concept of place value. He would see the number 36 as a collection of 36 ones. At this point, he should be able to put out a collection of 36 counters to show the number 36. Even though he has an understanding of cardinality, he would most likely count all the counters one by one.

With repeated practice of breaking down and building up of numbers, the child would begin to recognise the number relations in the number 36. He would be able to group his counters into three groups of ten, with six loose counters. He would then count in tens (20, 20, 30), and then count on 6 (31, 32, 33, 34, 35, 36).
ACTIVITY 2
Give children a collection of counters.
- Make sure the quantity is a two-digit amount.
- Watch how the children count.
  - Do they count in ones?
  - Do they make groups of counters?
  - If so, do they count the groups from one each time?
  - Do they use accelerated counting, and count in 2s, 5s or 10’s?

Commentary

Children need lots of practice in breaking down and building up numbers. It is essential that teachers provide a variety of activities, using different apparatus, to help them think about the best way to count larger numbers. It is a good idea to demonstrate how tricky it can be to count items when there are more than ten of them. You can model using one to one correspondence as you count 23 counters, but pretend to become confused about which counters have or have not been counted. You can re-start the counting a few times, and then talk about how you are becoming confused because there are too many counters to count easily, and it is taking too long to count in this way. You could then ask the children if they can think of a better way of counting. This will allow the children to come up with the idea of smart counting on their own, and they can provide suggestions about how to count 23 counters. Some children might suggest counting in 2s up to 22 and then counting on 1 to 23. Others may suggest counting in 5s up to 20, and then counting on to 23. Still others may count in 10s to 20 and then adding on 3 to make 23. You can use these different suggestions to help you see the various levels of understanding shown by the children in your class. This helps you in you assessment of children, and provides you with guidance in terms of how to plan and structure future lessons in order to meet the needs of all the children in your class.

Recognising and addressing children’s errors

As a responsible teacher of mathematics, you need to deal with learners’ errors when you encounter them. You might not always be able to address an error on the spot because it may be too time consuming in the context of a lesson where time is limited. However, you should always make a point of speaking to learners who hold misconceptions that will interfere with their mathematical learning. This is why it is so important that you understand the progression of learning within the development of number concept. This will help you to identify children’s misconceptions, and to provide them with a variety of playful opportunities to clarify their understanding of number.
ACTIVITY 3

Watch this video clip (11.58 minutes) of learners playing a game using a number track and number cards.

In this video the teacher works with learners who are playing a game that involves picking a card that gives a number of steps they can take across a river. The ‘river’ is shown by the number track – the track has 10 steps, so it is called the “Ten step game”.

After watching the video reflect on the following questions:

• What is the error that learners make and that the teacher addresses?

• How does the teacher address the error?

• Have you seen learners in your class making the same error and if so, how did you address it?

Commentary

The error that some of the learners make is that they do not move forwards when they count steps on the number track. For the first count, they stay on the same number. They just jump on the block which they are on, and then they start to move forwards. The teacher asks them to think about how they are counting every time, and she corrects them and shows them the right way to jump each time. She lets them try again and again, until they begin to make the correct moves.

The teacher only gives verbal instructions. She could have done drawings of number lines to show the jumps, since the jumps on the number track are like jumps on a number line. The number track is meant to help learners learn how to jump on a number line. Maybe you have done this with your class? Show them the jumps with little arrows to they can see how to move forwards every jump, starting from the first one. For example, if my counter is on 5 and I must jump 3 times, I will jump to 6, then to 7 and then to 8.
Check your understanding: Multiple choice

1) Smart counting:
   A) involves counting a collection of objects in ones.
   B) means finding the fastest and most accurate way to count a collection of objects.
   C) must always be silent counting.

2) Breaking down and building up numbers:
   A) Can be done in a number of different ways.
   B) Should only be done in tens and ones.
   C) Is part of rote counting.

3) An understanding of place value means that:
   A) Children can represent 15 using 10 blocks and 5 blocks.
   B) Children can count in tens and ones.
   C) Both A and B.

4) Children's errors:
   A) Can only be corrected through punishment.
   B) Are an opportunity for assessment and further teaching and learning.
   C) Are caused by carelessness and can be ignored.

REFLECTION

• Reflect on your own experience of counting when you were a child.
• Think about whether you used rote counting or rational counting.
• Consider the counting that happens in your classroom. Are children counting with understanding?
• Write down ways to improve the counting activities in your maths lessons.

Well done you have completed Lesson 7.
As we have seen from our previous lessons on Emergent Number and Smart Counting, children need to develop a sound understanding of number, recognising the ‘how manyness’ of a number and its relationships to other numbers. They need to extend their knowledge beyond an ability to identify numerals and number names, as they learn to break down and build up numbers. In this lesson we will therefore continue to investigate the concept of place value as we look at the equal exchange of numbers.

**What you will learn in this lesson**

- Playful opportunities to learn about equal exchange
- Equal exchange using money
- Equal exchange leading to an understanding of place value

As we saw in our lessons on play, games and playful activities help children to learn about maths in context, and to develop a greater depth of understanding. Have you ever watched children playing ‘Shop Shop’? They typically have a shopkeeper and a customer, and they exchange items for pretend money. In this game, children are assigning a specific value (accurate or not) to items, and then exchanging those items for money of an equal value. Children perform similar exchanges when they play ‘Swaps’. Many children create collections of stickers, marbles or even erasers, and then they swap with their friends. In this bartering process they establish their own system of determining the value of items, which may well be arbitrary, but which becomes an accepted norm in the group of friends. For example, children may decide that two shiny stickers are worth one furry sticker.

These playful opportunities are an important part of children learning about value. Children begin to determine what gives something its value, and to make sure that their exchanges are fair and balanced. The games that children play put the concept of equal exchange into context, and help them to verbalise their reasons for their exchanges. This will be of benefit to them as they extend their understanding of number, moving on to learning about place value and computations.

**ACTIVITY 1**

Observe the games that the children in your class play.

- Do they play games that involve bartering or equal exchange?
- If so, describe these games and consider what the children are learning from them.
- If not, describes games that you could introduce to them, explaining why you think these games have learning value.

**Commentary**

There are different types of playful activities that can be used in your classroom. Consider how you might use each of these types to encourage children to learn more about equal exchange and to develop their mathematical language.
**Equal exchange with money**

In maths, there are a number of different topics, concepts and skills that are covered. Very often, teachers approach these separately, teaching each one on its own, before moving on to the next idea. Unfortunately, this approach doesn’t allow children to see the inter-related nature of these topics, skills and concepts. As previously mentioned, the more children know about one number, the more they understand about that number’s relationship to other numbers. In the same way, a topic such as Money should not be taught in isolation. If the concept of Money is only taught for a few lessons before the teacher moves on to a different topic, then the children are likely to forget what they have learnt. It is a good idea to use what children know about notes and coins to help them develop their understanding of number, as seen in our Smart Counting lessons.

Children can use R10 notes to help them count in tens or they can use R2 coins to help them count in twos. These examples will help children see the R10 note as a group of ten, recognising that even though it is only one note, it has the same value as ten R1 coins. This will encourage children to progress from their initial stage of counting all.

**ACTIVITY 2**

Watch the video, “Equal exchange Part 1 (What is it?)”, (4.42 minutes) showing a class practicing equal exchange with money.

- What is the teacher getting the children to notice?
- Why couldn’t the children exchange R50 for R25?
- What other resources could you use in your classroom to teach this concept?
**Commentary**

In this activity children were learning that the number of notes they had did not necessarily relate to the value of the money. For example, a child holding five R10 notes has a higher quantity of notes, but a lower value, than a child holding one R100 note. It is important for children to realise that we can show the same value in different ways. This will help them as they learn about place value and as they begin to use this knowledge in their calculations.

Make sure that you continue to develop mathematical language by modelling for the children, and by asking them questions to get them to verbalise for themselves. Many teachers believe that classrooms should be silent in order for children to learn, but actually children need to be given the opportunity to talk to both the teacher and to their peers. Children can learn from each other as they share their ideas and compare what they have done.

**What is place value?**

Place value is an essential aspect of number concept that is developed throughout the primary school. Children who have a sound understanding of place value will develop a firm foundation for flexible methods of computation. Added to this, children who are given opportunities to develop their own methods of solving problems will have a better understanding of number and of place value.

### ACTIVITY 3

It is important to assess the level of understanding of place value that the children in your class have reached.

- Give the children a collection of multifix cubes, asking them to make 5 towers of 10 cubes.
- Ask them to add 3 more cubes, and then ask: ‘How many cubes do we have now?’
- Observe what the children do, and repeat with other, similar examples.
Commentary

Observation is necessary for accurate assessment. Teachers need to watch to see how the children in their class solve problems, and listen to the way that they verbalise their thinking. For example, in the above activity, if a child says that he has 8 cubes, it means that he has counted the towers of ten as 1 each. This tells the teacher that the child has not yet developed a sound number concept as he is using one to one correspondence to match one number name with one cube or one tower of cubes. Equally, if a child counts in ones from the beginning, even though they made the towers of ten themselves, the teacher learns that he is still counting all. The teacher can then modify her future teaching to help him progress to smart counting.

Check your understanding: True or false?

1) Bartering helps children to begin to understand the value of items.
2) Topics like Money should be taught in isolation so that children can focus their attention.
3) Place value is best learnt by the teacher showing the children examples.
4) Observation is an essential part of assessment.

REFLECTION

- Reflect on your experience of learning about place value. Did you just learn facts off by heart, or did you understand the concept?
- Think about how you are teaching place value in your own class. Do you allow children the opportunity to construct their own understanding?

Well done you have completed Lesson 8.
In this lesson, we will continue to explore the idea of equal exchange. We will further our understanding by learning more about place value. We will discuss the base ten system that we use in maths, and we will look at how we can exchange units for tens.

What you will learn in this lesson

- Base ten system
- The language of the base ten system
- Equal exchange of units for tens

Base ten system

As we have looked at Emergent Number and Smart Counting in our previous lessons, it has become clear that developing children’s number concept should not be hurried. Children need time to construct their own system of ones. This means that they understand a number like 55 as 55 ones, not 50 and 5. In order for them to understand the number 55 as 50 and 5, the children must first construct a system of tens.

This system of tens is known as a base ten system, and it uses only ten symbols (including a symbol for zero). This base ten system allows us to represent any number we choose, by using these ten numerals and the concept of place value.

5000 + 100 + 40 + 2 = 5142

ACTIVITY 1

Write the number 6824 in the middle of a piece of paper.
- Around the number, write down everything you know about this number.
- Use what you have learnt about rote counting, smart counting, breaking down and building up numbers, and number relationships to help you write as many number facts as possible.
Commentary

We may look at a number like 6824, and wonder how much we could possibly say about it. However, once we start thinking about what we know about numbers, we realise that there are many number facts that we could mention. For example, we could break down the number into 6000 + 800 + 20 + 4, and a variety of other combinations of numbers. We could also say that 6824 is 24 more than 6800 and only 76 less than 6900. We could also say that half of 6824 is 3412.

Our understanding of numbers as part of a base ten system help us to recognise the relationships between numbers beyond what we could do if we looked at the same number using a system of ones. Using anchor numbers like 2, 5 or 10 allow us to group quantities together in different ways, making it easier for us to identify many different methods of breaking down and building up numbers.

The language of the base ten system

In place value, the position of the numeral determines its value. The same numeral can represent different values, depending on its position. Place value is one of the most difficult concepts for young children to understand, and so teachers need to provide many opportunities for children to develop their understanding. As we discussed in our lessons on Smart Counting lessons, children need to see numbers as being connected to one another, not in isolation. This is why teachers should give children plenty of time to practice breaking down and building up of numbers. By being involved in a variety of playful opportunities, children are able to construct their knowledge of the base ten system and place value for themselves.

As children further their understanding, they also need to develop the correct mathematical language. This is linked to their knowledge of smart counting, helping children to see the grouping by tens concept. For example, if a child was using smart counting to help him talk about the number 53, he might say “There are five tens and 3 ones”. This is base ten language. As children gain confidence in identifying numbers like this, they will associate base-ten language with ordinary language, recognising that five tens and three ones is the same as fifty-three.

ACTIVITY 2

Think about what games and playful activities you could use to get children to talk about two-digit numbers.

• What resources could you use?
• What questions should you ask?

Commentary

There are number of ways that children can learn about two-digit numbers, and develop the maths language associated with them. Teachers could write a two-digit number on the board, and children could work in pairs to think of ways to break down the number. This will give children a chance to practice their smart counting, as well as to think about two-digit numbers in different ways. If children understand how to construct a number, they have a better sense of the number in terms of its relationship to other numbers.

Working in pairs or groups is a good way for children develop their maths language. They are often more confident sharing their ideas with their peers, and they also learn to mimic the way that their friends express their thoughts. Teachers therefore do need to be observant and listen carefully to these conversations to ensure that children are learning how to use the maths language correctly.
Equal exchange of units for tens

In order for children to successfully use a system of tens (rather than a system of ones), they need to actively participate in making an equal exchange of units for tens. Children need to physically see that they have taken ten items and exchanged them for one item. While we discussed using money in the previous lesson, this is only one option. For learners who find the shift to a system of tens a little tricky, money may not be the best resource to use as you can't actually see how ten R1 coins make up one R10 note. Notes and coins are completely different, and you can't put the R1 coins together to make a R10 note. However, ten frames and base ten blocks are both extremely useful resources to help children exchange units for tens.

When using a ten frame to exchange units for tens, children can easily see that a complete ten frame is a group of ten. It is important that children use the correct maths language to help them explain what they notice. Children could say "I had 9 units, and then I added one more. Now I have one ten". They might also refer to their ten as a "group of ten".

Children can be given a collection of the base ten units and the base ten rods, and asked to make a two-digit number. Teachers should observe whether the children are able to create the number by using smart counting, or whether they are still counting all. It is important to help children realise that ten unit blocks line up exactly next to the rod, clearly showing that ten units makes one rod, or one ten.

ACTIVITY 3

Watch the video, “Equal exchange Part 2 (Units to tens)” (5.38 minutes), showing a class using practicing equal exchange with base ten blocks.

- What is the teacher getting the children to notice?
- How does the teacher get the children to use the base ten blocks?
- What was the important aspect that the teacher wanted the children to learn from the activity by the end of the video?

Commentary

It is important for children to physically see that ten units are exactly the same as one rod. By placing the unit blocks next to or on top of the rod, they can easily see that we can exchange one rod for ten units. They will also realise that it is easier to work with one rod (or one ten) rather than try to manage the ten loose units. Even though it seems obvious to us as adults, children need to go through the process of matching the unit blocks to the rod so that they can construct this understanding for themselves. By doing it for themselves, they are also using the maths language as they verbalise the process.

In the video, the teacher is encouraging the children to resist counting each demarcation of the rods, but rather to recognise that one rod is one ten. She is trying to help the children progress beyond counting in ones, and to start using smart counting as an easier and more efficient
method. It is clear from this video that cardinality, smart counting and equal exchange are all inter-connected, and that the knowledge and skills related to these concepts need to be developed and practiced continuously.

**Check your understanding: Multiple choice**

1) When children work with a system of ones:
   - A) They would see 71 and 70 and 1.
   - B) They construct the number by counting all.
   - C) They understand place value.

2) In place value:
   - A) The position of the numeral determines its value.
   - B) The same numeral can represent different values, depending on its position.
   - C) Both A and B

3) During lessons, conversations between children:
   - A) Can be valuable learning opportunities
   - B) Are disruptive and unhelpful.
   - C) Should be kept to a minimum.

4) When learning about equal exchange:
   - A) Children should observe the teacher demonstrating the exchange.
   - B) Children find it easy to grasp the concept.
   - C) Children need to construct their own understanding by physically making the exchanges themselves.

**REFLECTION**

- Think about your maths lessons when you were at school. Were you an active participant in the lessons or were you an observer?
- Write a personal goal in relation to how you want to teach maths to children in your own class.

**Well done you have completed Lesson 9.**
As we have seen from our previous lessons on equal exchange, children need multiple opportunities to practice what they have learnt about building up and breaking down numbers, and place value. Therefore, in this lesson we will focus on getting children to practice equal exchange by exchanging tens for hundreds. You will notice that there are similarities to what we covered in the previous lesson, and this is intended to help children consolidate their understanding. Teachers need to point out the similarities to children to help them recognise that our number system consists of patterns that help us to simplify our mathematical knowledge and calculations.

**What you will learn in this lesson**
- Pattern recognition
- Equal exchange tens for hundreds
- Using Flard cards to practice equal exchange

**Pattern recognition**
Once children have established a sound understanding of our base ten numeration system, they need to consistently practice what they have learnt. As they play games, actively participate in lessons and verbalise their thinking, they will both consolidate and extend their understanding.

As people, we tend to like categorising and classifying things into groups, and to look for similarities (rather than differences) between these items. This is likely to be due to the fact that we like to look for patterns and order amongst things. In mathematics the idea of a set is used to talk about a group of items sharing a common characteristic. A set is a collection of objects that belong together. Pattern recognition is part of mathematical reasoning. We use pattern recognition when we decide whether or not numbers go into the same set. One of the benefits of pattern recognition is that, once we have decided on a pattern to form a set, we then don’t need to list all of the numbers to go in that set. For example, if we decided to create a set of even numbers, we wouldn’t need to list all the possible even numbers. However, knowing the pattern then helps us to work out that 898 would go in the set of even numbers, without us having to count up to that number.

**ACTIVITY 1**
Look at the picture of the sucker sticks below.
- How would you expect children to group the sucker sticks below?

```
[Image of sucker sticks]
```
- Why do you think they would have grouped the sticks in this way?
- How could you develop pattern recognition with the children in your class? Think of a variety of ways that don’t all have to be related to mathematics.
Commentary

Children can practice identifying patterns in many different ways. Teachers can provide opportunities for short pair or group discussions or activities in between lessons, or as a brain break between activities. The teacher could stand in front of the class and show them three items. Any items from the class will do, as long as the teacher has identified reasons for these three items to be grouped together. She could then ask the children to discuss in their groups or pairs what other items could possibly be added to the group. Children will then be able to discuss the criteria that could cause the three items to be part of the same group, and then use this criteria to select another item from the class to add to the group. For example, the teacher may have selected the dustbin, a lunch box and a base ten block. The children may consider colour, shape and function as reasons for the items to be grouped together. Hopefully, children may then suggest adding something like a juice bottle to the group because all the items are made from plastic.

Equal exchange of tens for hundreds

As we learnt in the previous section, pattern recognition helps us to simplify our mathematical knowledge and calculations. The identification of patterns will help children to extend their knowledge of place value to include exchanging tens for hundreds. In much the same way as they learnt to exchange units for tens, children will be able to exchange tens for hundreds. Once again, they need to physically see that they have taken ten rods and exchanged them for one flat. Children can actually place the rods on top of the flat to see that the flat is made up of ten rods stuck together. Encourage children to talk about how much quicker and easier it is to pick up the one flat than the ten rods.

Pattern recognition also helps children to develop a conceptual understanding of equal exchange. When they work with base ten blocks, and lay them out, they can easily identify the patterns that they see. This helps them to work out and understand problems involving equal exchange, rather than having to remember number facts off by heart.

<table>
<thead>
<tr>
<th>Units</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>+1</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tens</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>+10</td>
<td>100</td>
</tr>
<tr>
<td>Hundreds</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
<td>800</td>
<td>900</td>
<td>+100</td>
<td>1000</td>
</tr>
</tbody>
</table>

ACTIVITY 2

Watch the video, “Equal exchange Part 3 (Tens to hundreds)” (3.12 minutes), showing a class practicing equal exchange with base ten blocks.

• What is the teacher getting the children to notice?
• Why did the teacher build the number 110 twice?
• Ask children in your class to build three-digit numbers with base ten blocks, and write down your observations of how they completed the tasks.
Commentary

It is important for children to physically see that ten rods are exactly the same as one flat. By placing the rods next to or on top of the flat, they can easily see that we can exchange one flat for ten rods. They will also realise that it is easier to work with one flat (or one hundred) rather than try to manage the ten loose rods. As previously mentioned, children will need to physically match the rods to the flat so that they can continue extending their understanding of equal exchange.

By practicing equal exchange of tens for hundreds in the same way as they did with units for tens, children will further develop their ability to recognise patterns. They will also be able to use these patterns to work through mathematical problems quickly and easily, without having to construct their knowledge from scratch every time.

Using flard cards to practice equal exchange

We can use flard card to make the number 439. The number would be written on three separate cards, which are placed one behind the other to look like this:

```
400  30  9
```

Children need to be given the opportunity to build up and break down numbers using the cards. As they separate the number into the three cards, they can see that the number 4 in the number 439 actually has the value 400. In the same way they can see that the 3 has a value of 30, and that the 9 is simply 9 units.

It is a good idea to ask children questions about three digit numbers in order to get them to talk about the relationship between the numbers, and to use the maths language. As they talk about the numbers, they will extend their understanding about how the position of the numeral determines its value. Children can play a calculator game called “ZAP” to help them develop their knowledge and understanding. One player calls out a number for the other players to enter onto their calculator displays (e.g. 4 789). The player then says “ZAP the 8”, which means that the other players must replace the 8 with the digit 0, using one operation (i.e. to change it into 4 709). The player who is the quickest to decide on how to ZAP the given digit could call out the next number.

ACTIVITY 3

A learner in your Grade 2 class claims that 15 < 9. When asked why she has given this answer, the learner reasons that 9 is the biggest digit.

• What is wrong with the learner’s reasoning in the above explanation?
• What is the concept embedded in the question?
• What concept is interfering with the learner’s reasoning in this instance?
• In what way is this learners’ understanding different to what the teacher expects?

Commentary

The reasoning the learner gives for her answer shows that she has not yet fully understood the concept of place value. She drew on her understanding of whole number concept to compare the numbers 1, 5 and 9. The learner needs to expand her number concept so that she can correctly read 15 as 1 ten plus 5 units which is 15 and which is bigger than 9. Is a learner reasons in this way the teacher should allow the learner to make displays of the two numbers using counters to enable her to see that 15 is greater than 9.
**Check your understanding: True or false?**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Humans tend to enjoy finding similarities between items, and using these to establish patterns.</td>
<td>True</td>
</tr>
<tr>
<td>2) Pattern recognition in maths can only be developed through mathematical activities.</td>
<td>False</td>
</tr>
<tr>
<td>3) Pattern recognition can help children to construct their own understanding of mathematical concepts.</td>
<td>True</td>
</tr>
<tr>
<td>4) Games have limited value in teaching children about equal sharing.</td>
<td>False</td>
</tr>
</tbody>
</table>

**REFLECTION**

- Think about your assessment practices in your maths lessons. Do you continuously assess children, or do you just look at the book in their books?
- Write a personal goal in relation to how you plan to assess children going forward.

Well done you have completed Lesson 10.
In Part 2, you will be looking at addition and subtraction. We will discuss the importance of mathematical language, and we will investigate patterns, the use of resources and different strategies used in the operations. We will also learn more about using addition and subtraction in problems involving money. Finally, we will spend time exploring addition and subtraction as inverse operations.
In this lesson, we will introduce the concept of addition. We will begin by looking at Mathematical language, and its importance in the development of a sound number concept. We will then revisit the notion of play, considering how addition can be taught in context. We will also discuss assessment, and think about learner errors, so that we can further our understanding of how children learn.

What you will learn in this lesson
• The importance of Mathematical language
• Addition in context – learning through play
• Using assessment and learner errors to inform teaching

Mathematical language
It is important to provide multiple opportunities for children to be exposed to mathematical language. The knowledge and understanding of appropriate mathematical vocabulary helps children to express their thoughts and ideas about mathematical concepts. This ability to use mathematical language accurately is an essential part of the learning process. Children will use this language to participate in and complete tasks, and to respond to questions.

In some instances, there is a difference between a child’s social (or everyday) understanding of a word and his academic (or mathematical) understanding of a word. An example of this would be the word ‘table’. In a social context, the word describes a wooden piece of furniture at which we might sit to do our homework. In an academic context, the word refers to the way in which information can be organized and displayed. Teachers need to help children to make connections between their prior knowledge and the new mathematical language. One way in which to do this is to focus on the use of resources as a way to introduce new concepts. Children need to be encouraged to use language to make connections between concrete apparatus, pictorial representations and abstract mathematical symbols. As a teacher, it is necessary for you to create a safe learning environment and to model the correct use of mathematical language so that children develop their ability to verbalise their solution methods.
ACTIVITY 1

Watch the video “How many altogether” (2:03 minutes) to see how the teacher teaches the concept of addition to her Grade 1 class.

• What specific mathematical language can you identify in the video?

• Write a list of any other words that you can think of that relate to the concept of addition.

Commentary

There are many words relating to the concept of addition that you may have identified: Add, addition, plus, equals, altogether, more than, increase, total, groups of, sum.

It is important to ensure that children recognise these words, and that they use them correctly. In our everyday language, we have developed habits of using words such as ‘sum’ to refer to any mathematical problem. This is not the correct use of the word, as in a mathematical context the ‘sum’ of a number refers specifically to the total amount resulting from the addition of two or more quantities. Therefore, when we use the word ‘sum’ to refer to a subtraction problem, we are telling children that the vocabulary used in mathematical contexts can be used interchangeably. We cannot then be surprised when children make statements such as my half is bigger than your half. Children need to understand that the words used in maths have specific meanings, and they should use them appropriately. This will prevent confusion as they progress throughout their schooling and help them to respond to questions accurately. Ultimately, a sound understanding of mathematical language is the first step to improving learner performance in mathematics.

Addition in context

When we introduce any operation, we should begin by using numbers in a real context. To do this, we can tell stories that lead to the addition or subtraction of numbers. This makes it clear to the learners what they need to do, and it also lays a foundation for their own problem-solving later, when they will have to read and interpret word problems. These stories help children to see mathematics as being relevant to their everyday lives, rather than something that is only done in a Maths lesson at school.

In Maths, we refer to children’s ability to solve problems effectively as mathematical fluency. This means that children are able to flexibly select the best strategy to help them solve problems quickly and accurately. In order to improve children’s mathematical fluency, we need to allow them time to learn number facts, such as their number bonds, until they are able to recall these with ease. However, this is not just rote learning of number facts. Encouraging children to verbalise their methods helps them to develop their conceptual understanding, which in turn will enable to face problem-solving tasks with greater confidence.
ACTIVITY 2
• Give an example of an addition story that would be relevant to the children in your class.

Commentary
Think about what the children in your class may be interested in, such as marbles, sweets or balloons. You may even be able to include items related to the topic you are covering in class at the moment. An addition story needs to be simple and clear, so that children can work out what needs to be done to solve the problem. Initially you may even keep to the same sentence structure, such as:

Nosipho has 5 balloons.
Her brother gives her 3 more balloons.
How many balloons does she have now?

Children need to learn to identify the important information in the addition story, and to use this to solve the problem. Once they have become confident in this, you could increase the complexity of the stories.

Learner Errors
In order to help children become confident in their mathematical fluency, and to effectively reason and solve problems, we need to continuously assess their learning and understanding. Some assessments may take the form of written tasks; however, it is more meaningful to assess through continuous observation, and a combination of practical and oral tasks as well. If we rely on just one written task to assess children, then we do not take in consideration any contextual factors that may influence a child’s performance. It is important to remember that assessment is an emotional process, and that many children experience anxiety over formal assessment tasks.

To provide rich opportunities for observation in class, give your learners time to play games and while you do so watch the way they they:
• use mathematical language
• give mathematical answers
• deal with errors they or their partners in the game make.
It is important to use your assessment to improve the teaching and learning that takes place in your class. Through careful observation and discussion with the children, it is possible to determine where their misconceptions may fall. You can then use this knowledge to redirect your future lessons, and to rectify the children’s misconceptions.

**ACTIVITY 3**

Look at the problem below.

\[
30 + 500 + 7 = 1s
\]

- What do you think the child's misconception might be?
- What could you do to rectify this misconception?

**Commentary**

In this example, the child has added the digits without considering their value. He has disregarded the zeros, assuming a nil value for those, and then added the 3, 5 and 7 as if they were all ones. The child does not have a sound understanding of place value, and additional work needs to be done to help him to move on from viewing numbers as a collection of ones to a system of tens. Concrete resources such as base ten blocks can be used to help him recognise that 10 ones is the same as 1 ten, and that 10 tens is the same as 1 hundred.
Check your understanding: True or False?

1) The word 'sum' refers to all mathematical problems given to children.

2) Stories are used to make Maths more entertaining for young children.

3) Written assessment tasks are the most valuable way to assess children.

4) Mathematical language plays an important role in the assessment of children as well as in rectifying misconceptions.

REFLECTION

• Reflect on your use of mathematical language in the classroom.
• How could you model the correct use of specific vocabulary?
• Write a personal goal in relation to the promotion of mathematical language in your classroom.

Well done you have completed Lesson 1.
In this lesson, we will focus on patterns. Patterns are part of our everyday lives, and it is human nature to seek out and identify patterns. We will look at types of patterns, and then consider how patterns are used in the teaching and learning of addition. We will also discuss number lines and the various strategies that children can use when solving addition problems.

**What you will learn in this lesson**

- Why are patterns important?
- Patterns and number lines
- Addition strategies

**Why are patterns important?**

Patterns help us to simplify our world by classifying different objects and situations, allowing us to create a sense of order into our complex environment. The ability to recognise the same pattern in different situations enables us to transfer our knowledge of one context to another context without studying each one separately. Order, regularity and sequence are at the heart of Mathematics, and children need to develop their ability to find and recognise patterns.

Children can be exposed to patterns in everyday life, such as the patterns found in curtains, tiles, windows and bricks. Children can also physically experience patterns by playing hopscotch, clapping games, and by identifying visual, auditory and tactile patterns.

Patterns play a valuable role in the teaching and learning of number concept. Unless children can recognize and apply number patterns, their only tool for problem solving is counting. So, if we want to help children progress from solving problems by counting all, we need to help them identify numerical patterns that will improve their mathematical fluency. The images show a teacher teaching about patterns using number bonds. In the second image the teacher has removed the bonds of 7 and is discussing the pattern made by doing so.
ACTIVITY 1
Think about the patterns you encounter in your everyday life.
• Describe a visual pattern that you can see in your classroom.
• Describe an auditory pattern that you may have heard recently.
• Describe a tactile pattern that you could get the children in your class to experience.

Commentary
Activities where children discover patterns, and the rules which are part of these patterns, lay a foundation for algebra in later schooling. Getting children to identify a variety of patterns, and to talk about the patterns, helps them to recognise the rules that they have created in order to establish the pattern. Even when the patterns may be auditory or tactile in nature, or comprised of colours, shapes, different sized objects or any other visual possibility, the identification of the pattern lays the foundation for learning about number patterns. Once children are able to recognise and describe visual, auditory and tactile patterns, they will then be able to transfer these skills to activities involving number patterns. Pattern recognition is part of mathematical reasoning. Activities that involve deciding whether or not numbers fall into the same set call on this pattern recognition skill.

Patterns and number lines
Teachers can continue using patterns to help children develop their mathematical fluency when they focus on the use of number lines. Number lines are useful as they show the position of numbers in relation to one another. This gives children opportunities to verbalise their understanding of number, using the appropriate mathematical vocabulary. Number lines can also be used to promote ordinal numbers and counting skills. Children can use a number line to count forwards and backwards, in ones or in multiples, as well as counting on or back from a given number. Children can discuss the patterns that they discover as they use number lines to addition and subtraction problems.

Young children can start to experience placing numbers in sequence on a number line using number cards as shown in the following images. When doing such an activity the teacher should ask questions about the relative sizes of the numbers and how to make the decision of what number is placed where on the number line.

When children are able to use patterns to help them solve problems, this means that they are able to rely on previously constructed knowledge and understanding in the process of working out their solutions. The fact that they do not need to re-construct their understanding for each new problem means that they are working with greater efficiency. As previously mentioned, our goal as mathematics teachers is to help children to flexibly select the best strategy to help them solve problems quickly and accurately. If children are able to do this, using a strong foundation of mathematical language, they will be well on their way to confidently approaching tasks involving reasoning and problem-solving.
ACTIVITY 2

Number Lines and Number Patterns

Watch the video “Number Lines and Number Patterns” (4:40 minutes) to see how the teacher uses the idea of patterns to help children count in 2s on a number line.

• Describe the patterns identified by the children in the video.
• Can you see another pattern that the children didn’t identify?
• How would you help children to see the patterns involved in counting in 2s on the number line?

Commentary

By identifying the patterns involved in counting in 2s on the number line, the children are creating a foundation for the idea of repeated addition. They can see that the jumps on the number line are the same as adding 2 each time. The teacher also discusses the idea that there is pattern of coloured buttons. It is possible to elaborate on this pattern, in that the order of the three colours is reversed for the second group of three buttons. A pattern that was not identified in the video was that the placement of the buttons skips one number each time. This could also have led to a discussion on odd and even numbers.

The teacher tries to model the use of mathematical language, by rephrasing the answers given to her by the children. This is an important part of the process, as children need a great deal of practice in order to be able to verbalise their ideas confidently. When the teacher repeats or rephrases what the children have said, she is showing them how to express themselves clearly, using the appropriate terminology.

Addition strategies

As part of the continuous assessment process, teachers can use observation, practical activities and oral tasks to help them to recognise some of the strategies that children use to solve problems. Many children struggle to make the shift from solving problems by counting to using calculation strategies. It is therefore essential that teachers spend time drawing children’s attention to the different strategies and to model these in their teaching. Teachers should be aware of the range of available methods, so that they can support the children who use these various methods. Children can use the appropriate mathematical language to discuss and share their methods, and this can encourage other children to try out the methods. With practice, children will then begin to select appropriate strategies for the problems they are given.
Near doubles

8 + 8
children will know that the answer is 16 because they know that 8 doubled is 16

Doubles facts

Calculation strategies

Bridging through 10

Compensation

7 + 5
Children know that 7 + 7 is 14 and they take away 2 from 14

8 + 7
take 2 from 7 and add it to 8 in order to make 10 and add 5 to 10. The answer is 15

9 + 6
add 1 to 9 in order to have 10 and add 6 to 10, subsequently, they subtract 1 from 16. The answer is 15

ACTIVITY 3
Look at the addition strategies discussed above.

• Select one of the strategies and describe the pattern that is evident in the use of this strategy.

• How could you help children identify the patterns in addition strategies?

Commentary
When children move on to using calculation strategies rather than simply counting to solve problems, it is helpful highlight the patterns involved in these strategies. There is a tendency to teach calculation strategies as a method to be learnt by children. However, this tends to make children try to follow a ‘recipe’, which often results in errors because they do not fully grasp the strategy that they are using. If children are given opportunities to discover calculation strategies for themselves, and to verbalise the patterns involved in using these strategies, they are more likely to understand the process.
### Check your understanding: Multiple Choice

1) **Patterns:**
   - a) Create a sense of order.
   - b) Are pretty to look at.
   - c) Both of the above.

2) **Pattern recognition:**
   - a) Is only important for pre-number concepts.
   - b) Is a transferable skill.
   - c) Cannot be taught.

3) **Recognising patterns in number lines and addition strategies:**
   - a) Does not help children solve problems more efficiently.
   - b) Shows children the recipe for solving problems.
   - c) Helps children to understand what they are doing when they choose strategies to solve problems.

4) **It is important to talk about patterns because:**
   - a) It gives children an opportunity to be creative.
   - b) Children learn to verbalise their reasoning as they explain the rules of their pattern.
   - c) Children need to be able to speak confidently in class.

### REFLECTION

- Reflect on your experience of patterns in your everyday life.
- Think about whether or not you have explicitly addressed the idea of patterns in your class.
- Describe what you could do to increase the opportunities for pattern recognition in your lessons.

**Well done you have completed Lesson 2.**
In this lesson, we will look at the use of manipulatives in the solving of addition problems. We will first discuss the different types of manipulatives that could be used, and then consider the progression involved in the use of resources. Finally, we will investigate the use of ten frames in addition problems.

**What you will learn in this lesson**

- What are manipulatives?
- Addition and concrete resources
- Addition and ten frames

**What are manipulatives?**

Manipulatives are items used by teachers and children to represent abstract mathematical concepts. Research tells us that children use manipulatives to bridge the gap to an abstract understanding of number and mathematical operations. By physically handling concrete resources, children begin to make connections between the mathematical language, symbols, and pictorial representations. These connections lead children towards the development of mental images which will help them to solve problems in a more abstract way.
ACTIVITY 1
For each apparatus mentioned give one practical way in which that apparatus could be used in the teaching of addition.
1. Abacus
2. Base ten blocks and ten frames
3. Multifix cubes
4. Hundred squares
5. Number lines and number tracks

Commentary
An abacus is a useful way for learners to manipulate the beads and to observe the groups of five and ten. This helps them to develop a sense of these numbers as ‘anchor’ number, which then helps them to develop efficient strategies for solving problems. Base ten blocks can be used for simple addition of small numbers, as well as for addition of up to three-digit numbers. Multifix cubes are plastic blocks that can be stuck together and taken apart, and so these are useful to use in a variety of activities. Children can make groups of numbers, they can add cubes to represent an addition problem, and they can also make towers of ten to represent place value. Hundred squares can be used for counting on, as well as for demonstrations of bigger number addition and subtraction, using accelerated counting on or taking away. Number lines are a valuable resource to show jumps along a number line for early addition problems. Bigger jumps can also be shown on a number line, helping children to count in multiples. The use of number lines is very good in consolidating number concept.

Addition and concrete resources
As we have seen from our discussion on manipulatives, it is important for children to be introduced to a concept with the aid of concrete resources. When learning about addition, children can use a number of different items such as counters, cubes and beans. These resources provide a visual representation of number, helping children to establish connections between the abstract symbol, the mathematical language and the physical quantity. The resources will also enable the children to discover the notions of more than and less than as they physically position the items.

Once children have actively constructed their understanding of addition by using concrete resources, then they can progress to pictorial representations. Some children may need to begin by using the concrete resources, and then drawing these so that they have both representations visible at once. This will help them to make the shift from physical objects to the more abstract two-dimensional drawings. Other children may find that they can progress on to the pictorial representations without needing to see the drawings next to the concrete resources.

Once the children are confident with the use of pictures to represent addition problems, then they can progress to the abstract symbolic representation. It is important that teachers don’t move children on to this stage before they are ready.

ACTIVITY 2
Describe activities in which you can help children to develop their understanding of the bonds of ten.
• In the first activity, use concrete resources to help children.
• In the second activity, use a pictorial representation to represent the number bonds.
• In the third activity, focus on the symbolic or abstract representation of number bonds.
Commentary

There are many possibilities you could use as concrete resources when teaching children about the bonds of ten. One option is the number mat that was discussed in Part 1 Lesson 6. Children could take 10 counters and scatter them on the number mat. They would then record how many counters fell on the left-hand side of the line, and how many counters fell on the right-hand side of the line. They could record this in their book, before collecting up the counters and scattering them again. In doing this, the children will discover the different bonds of ten and create the different number bonds.

Then, the children could draw dots as a pictorial representation of the bonds of ten. This is a super way of helping children identify the pattern of the number bonds, and to create a mental picture for themselves.

This can then be further extended into the number bond ‘house’, which is a symbolic representation of the number bonds. Once again, this is useful for children to see the pattern of the bonds.

Addition and ten frames

As children develop their sense of number, they can learn to subitise by using ten frames. The layout of the ten frames makes it easy for children to see five or ten quickly. Children will also begin to determine numbers like 7, 8 or 9 more easily as they will recognise that the empty blocks on the ten frames help them to work out the number. A particular point to note is the fact that children may use addition or subtraction to work out missing numbers on a ten frame. For example, children may see 6 counters on the ten frame and count on to 10. They could also identify that they know that 10 – 6 = 4, so therefore the empty blocks equal 4.

It can easily be seen that ten frames are a particularly worthwhile concrete resource to use when learning about addition. They provide an opportunity for children to physically move counters on the frame, creating a visual representation of the bonds of ten. Children can verbalise what they see as they construct their understanding of the bonds of ten. By placing counters on a ten frame, it is easy to see how many more counters are needed to make ten.

ACTIVITY 3

Addition using Ten Frames

Watch the video “Addition using Ten Frames” (5:20 minutes) to see how the children use ten frames to help them solve addition problems.

• What did you notice about the way the first child positioned the counters on the ten frame?
• Why do you think the way the second child positioned the counters is better?
Commentary

In the video, the first child laid out the counters as shown below:

![Counter Layout 1]

The second child then adjusted the layout of the counters to look like the example shown below:

![Counter Layout 2]

Whilst this is not incorrect, it is easier to see the number 15 if the counters are positioned as the teacher demonstrated. This ties back to our discussions on subitising in Part 1, where we discovered that the layout of objects helps us to quickly determine a number without counting. With the full ten frame on the left, we can see this as a complete ten, and it lays the foundation for our work with place value tables where the tens are on the left and the ones are on the right.

Check your understanding: True or False

1) Concrete apparatus helps children create connections that will help them to work in a more abstract way.

2) Children’s understanding of concepts moves through a progression from concrete to symbolic to pictorial.

3) Number bonds are number facts that just need to be learnt off by heart with no need for manipulatives.

4) There are multiple ways to lay counters out on a ten frame, but some ways make it easier to work out the number.

REFLECTION

• Reflect on your use of manipulatives in your own classroom.

• What do you think the main challenges are / would be in terms of using manipulatives in your lessons?

• What do you think the main benefits are / would be in terms of using manipulatives in your lessons?

Well done you have completed Lesson 3.
In this lesson, we will continue investigating the use of manipulatives in the solving of addition problems. When working with larger numbers in addition, base ten blocks are a useful resource to help children understand the value of the numbers. However, as children become more confident in the solving of problems, a move towards using pictorial representations becomes an option. We will discuss both of these situations, before looking at some of the potential problems with using manipulatives in the classroom.

**What you will learn in this lesson**

- Addition and base ten blocks
- Pictorial representations
- Potential issues with manipulatives

Give learners lots of time to become familiar with the manipulatives you will use. Base ten blocks are particularly useful in the teaching of 3-digit number sense. Flard cards are also useful. They can also be used when teaching addition and subtraction of 2- and 3-digit numbers. Learners need to develop a sound number sense (discussed in Part 1) - this will begin when you introduce them to the numbers and it will be consolidated when they work carefully with numbers in the context of additive operations. It is always a good idea to allow learners time to play conversational games where they can develop their mathematical language. They can spend time showing and talking about numbers using manipulatives in preparation for using the manipulatives when doing operations.

**Addition and base ten blocks**

When adding larger numbers, children can use base ten blocks with a place value mat as concrete manipulatives. This is a useful way for children to physically see the process of equal exchange. They can lay out the tens and the ones on to the place value mat, and then add them together. In the images below, we can see that the children add 5 ones and 2 ones first. They then add 2 tens and 4 tens and 1 hundred to get a total of 167.
Once children have consolidated their understanding of adding larger numbers without regrouping, they will be able to use base ten blocks to solve problems involving regrouping.

Correct! We can exchange and make 1 hundred. How much do you have altogether?

I have 1 hundred, 2 tens and 9 ones.

ACTIVITY 1

Higher Number Addition using manipulatives

Watch the video “Higher Number Addition using manipulatives” (5:20 minutes) and see how the teacher encourages the children to use their base ten blocks.

- How do the children initially solve the problem 23 + 39?
- What is the ‘quick counting method’ that the teacher encourages the children to use?
- How does the use of base ten blocks help the children to develop their understanding of place value?
Commentary

In the video, the children need to solve an addition problem that involves regrouping. In order to solve the problem, the children are encouraged to use the ‘quick counting method’. This means that they exchange ten ones for one ten. The teacher is trying to get the children to practise equal exchange so that they can consolidate their understanding of place value. It is important for children to recognise that they can put ten ones together to make one ten, and that it is easier and quicker to count in tens than in ones.

Children need to have many opportunities to practice equal exchange. This is a difficult concept for children, and the use of physical resources help them to develop a better understanding. The use of manipulatives prepares children by showing them what exchange actually involves before they attempt to solve problems in an abstract way.

Pictorial representations

As children practice addition with larger numbers, it will become necessary for them to start drawing the representations of the numbers rather than try to manage increasing quantities of physical apparatus. This is a logical progression in their understanding and will further prepare them for the more abstract methods of solving problems.

Some children may need to have the concrete apparatus in front of them before drawing the resources, whilst others may be able to make the shift from concrete to pictorial more effortlessly. It is important for children to develop a quick and easy method of creating pictorial representations. Initially, children may try to draw perfect representations of the resources, but they need to learn to draw simple shapes or lines so that they can solve problems quickly.

A number line is another pictorial representation that is often used to represent numbers. To draw a number line correctly you need to choose an appropriate scale and your measurements must be accurate.

ACTIVITY 2

Look at the number line and addition problems below.

- How could you help children to use the number line to solve the problems?
- Why do you think a number line is counted as a pictorial representation?
Commentary

The children can draw jumps on the number line to show their addition. Initially they may be counting on from the first number, using a simple method of solving the problem. As their understanding of number progresses, they will begin to select other calculation strategies such as compensation or bridging the ten. If children were given the problem 138 + 6, they may choose to break up the 6 into 2 + 4 so that they could solve the problem as shown below:

In this way, the children can add 2 to get to 140, and then add on 4 to complete the calculation. They would do this because it is easier to use the multiples of ten as pivot numbers, rather than trying do equal exchange mentally.

Some potential issues with manipulatives

Many children find it difficult to move from the concrete representations of number to more abstract symbolic problems. This may be due to the fact that children become dependent on the concrete resource and rely heavily on the physical action involved in manipulating the items. Teachers may also underestimate children’s ability to solve problems in an abstract way, and so forget to encourage them to move beyond using the resources when appropriate.

Researchers believe that mathematical language is the key to bridging the gap between using concrete resources to solve problems and a more abstract method of solution. Children need to be encouraged to verbalise their actions as they manipulate the resources, and to consolidate their understanding of the concepts. By doing this, they are constructing mental images which can then replace the need for the physical resources.

As children begin working with larger numbers, it becomes even more necessary for them to shift to an abstract method of solving problems. Trying to use concrete resources to solve problems involving four-digit numbers can become problematic. At times, the number of resources required to construct large numbers becomes overwhelming, and results in more errors as items become misplaced or overlooked.

ACTIVITY 3

Look at the problem below:

345 + 587

• What resources do you think children should use to solve this problem?
• Why did you choose this resource rather than any other?

Commentary

When solving problems that involve large numbers, it becomes impractical to use certain concrete apparatus. For example, to use counters or multifix blocks for this problem would be difficult as there would be too many resources for the children to count or sort accurately. Base ten blocks would be more manageable as children would be able perform an equal exchange, and physically show the total amount. Children may also use a number line, as this would be an efficient method of solving a problem with large numbers. They could show jumps along the number line, from a simple method of counting on to the more advanced calculation strategies.
### Check your understanding: Multiple Choice

1) **Base ten blocks are:**
   - A) Useful for working with larger numbers.
   - B) Problematic when solving regrouping problems.
   - C) Both A and B

2) **It is important for children to:**
   - A) Count in ones when solving problems.
   - B) Solve problems without physically handling the base ten blocks.
   - C) Practise equal exchange by using concrete apparatus.

3) **Pictorial representations are:**
   - A) An unnecessary step in the progression of learning.
   - B) Easier to manage than large quantities of concrete resources.
   - C) Time consuming.

4) **Mathematical language is:**
   - A) The bridge between using concrete resources and solving problems abstractly.
   - B) Unimportant in the progression of children’s learning.
   - C) The bridge between confusion and understanding.

### REFLECTION

- Think about what you have noticed in terms of children’s progression of learning in Maths.
- What have you noticed about children’s ability to verbalise their actions and their level of understanding?
- What have you noticed about the amount concrete and pictorial resources used in your lessons?

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**Well done you have completed Lesson 4.**

**Answers:** 1) A, 2) C, 3) B, 4) A
In this lesson, we will discuss Money. This is an important topic, and children need much practice in order to confidently solve money problems. As teachers, we are often worried about how much time we have available to get through the curriculum, and so we focus more on getting children to solve money problems rather than on coin and note recognition. However, children will find money problems and practical activities extremely difficult if they are unsure about the notes and coins, so the recognition of these should be our starting point in this topic.

What you will learn in this lesson

• Coin and note recognition
• Addition and subtraction with money
• Practical activities

Coin and note recognition

When looking at coins and notes with your children, it is a good idea to make sure that children are actively involved by looking at resources, and describing what they see. If you just tell them what is on the coins and notes, they are likely to forget easily, whereas if they investigate for themselves, they are more likely to remember what they have learnt. The South African Reserve Bank and the South African Mint are both fantastic sources of information if you would like to find out more facts about the notes and coins we use.

In 2023, the South African Reserve Bank has revised the coins and notes in circulation and so there are quite a few changes to look out for. The 1c, 2c and 5c coins have not been in circulation for some time now, leaving only the 10c, 20, 50c, R1, R2 and R5 coins. The R5 coin now has a blue whale on it, the R1 coin has a protea, the 50c has a Knysna Turaco, and the 10c coin has a bee on it, which is the first time an insect has been represented on a South African coin.

The notes have been adapted to included increased security features, so that people can use the Look, Feel, Tilt strategy to check if the notes are fake. When you look at the notes, you can see the watermark and the numerals. There are certain features which are raised on the notes so that you can feel them, and these particularly help the blind and partially blind communities. When you tilt the notes, you will see a circle that spins and changes colour.

ACTIVITY 1

Research the South African coins and notes, and then brainstorm activities that you could use to help children recognise them.
Commentary

<table>
<thead>
<tr>
<th>Money Bingo</th>
<th>Children can play a bingo game where they identify the different coins or notes called out by the teacher.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have... who has... ?</td>
<td>Children get cards that say (for example): ‘I have R10, who has 50c?’ The person with the card that says ‘I have 50c ...’ would then have to read their card aloud. Pictorial representations of the notes and coins would help children recognise their features.</td>
</tr>
<tr>
<td>Snap game</td>
<td>Children could get cards with money amounts written on them, and then cards with coins or notes on them. Children could play snap where they match the cards.</td>
</tr>
<tr>
<td>Memory game</td>
<td>This would be much the same as the snap game, but the children would have to try recall the location of the cards that are placed face down on the desk.</td>
</tr>
<tr>
<td>Quiz</td>
<td>The teacher could call out features and children could hold up the correct coin or note that matches that feature.</td>
</tr>
</tbody>
</table>

**Addition and subtraction with money**

In the beginning, when first starting addition and subtraction with money, it is sensible to reinforce the idea of more than, less than and the same amount. You can hold up a coin, and ask **Who has more?**, **Who has less?** or **Who has the same amount as me?**. Children will need to recognise the value of their coins and hold up their coins in response to your questions.

Children also need to realise that an amount of money can be made up in different ways.

This is a challenging concept, and children will need multiple opportunities to consolidate their understanding. It is a worthwhile idea to gather a collection of priced items from supermarket specials leaflets. You can then use these to ask questions such as Show me how you will pay for this? and Who else paid in this way?. This gives children a chance to verbalise their selection of coins or notes, and to listen to the selections made by their friends.

As you can see from the videos shown, there are many opportunities to solve addition and subtraction problems using notes and coins. You could use the notes and coins in the place of base ten blocks, using these to practice equal exchange.
ACTIVITY 2

Money Matters (addition)"

Watch the video “Money Matters (addition)” (5:37 minutes) to see how children solve problems using notes and coins.

- Explain how the children use the notes and coins to solve the problem.
- How is this method similar to using base ten blocks as a resource?
- What does the video highlight about counting methods?

Commentary

The children use R10,00 notes and R1,00 coins in much the same way as they would use base ten blocks to represent tens and ones. They are able to use the notes and coins to add two quantities together, practicing equal exchange when they notice that they have eleven ones. The teacher encourages them to use mathematical language to verbalise their exchange, as they replace ten R1,00 coins with one R10,00 note.

It is evident that the children count in ones as they solve the problem. It is essential that we encourage children to move beyond counting in ones in order to solve problems more efficiently. As previously discussed in both Part 1 and earlier in Part 2, we need to model the different ways of getting to an answer. As teachers, we need to help children get used to counting on from the larger number, using bonds of ten and skip counting where appropriate.

Practical activities with money

Many teachers enjoy the opportunity to involve children in the transactions of buying, selling and the giving of change. This can be done by setting up a real-life shop in your classroom and bringing in items for the children to pretend to buy and sell. However, it is necessary to think about whether this is the most sensible option for you and your classroom. Setting up a real-life shop takes up a great deal of space and resources, which may be tricky to cope with in a classroom setting. It may be a better idea to use the priced items mentioned earlier in this lesson instead of real items. The pictures of items are easy to collect, and they take up less room in your classroom. You could use these pictures in a variety of ways:

- You could have loose pictures which children could select for themselves and place in a shopping trolley.
- You could paste a selection of pictures onto cardboard as a leaflet or a ‘shop window’, and then ask the children questions such as You have R10. Which items would you buy for your breakfast?

These are great ways to get children working with money in context. Children will need to think practically about how much money they have, and how much they can spend. It is possible to include an extra element by getting children to work out what change they will be given as well.
**ACTIVITY 3**

Describe an activity that you could use to get children to practise adding and subtraction with money.

- List the mathematical language that would be appropriate for this activity.
- Clarify the resources that you would use in this activity.

**Commentary**

For this activity, you might choose to use priced items, and tell children that they can go to the tuck shop to buy some items. You could have a collection of items to be bought, and you could ask questions to get the children thinking about how much they could afford to buy, and how much change they would receive.

You have R10,00

At the tuck shop you buy:

_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________

You spent: _________________________
Your change was: __________________

The mathematical vocabulary that might be relevant for this activity is: buy, spend, change, cost, afford, expensive, cheap, bargain, half price, discount. There are many options that you could include to reinforce your mathematical concepts. For example, you might add a sign to your ‘shop window’ that says, *Half Price Sale* or *3 for the price of 2!*. These additions would allow the children to practice halving or doubling within the context of the money problems.
Check your understanding: True or False

1) Many of the South African coins and notes were revised in 2023.

2) Coin and note recognition is an essential first step in learning about Money.

3) Coins and notes should only be dealt with in word problems.

4) Mathematical language is not important in the topic Money.

REFLECTION

• Reflect on your experience of teaching Money.
• Think about the kinds of practical activities you have / have not done in your classroom.
• Write a personal goal in relation to using resources to teaching Money.

Well done you have completed Lesson 5.
In this lesson, we will return to our consideration of mathematical language as we look at the concept of subtraction. Once again, we will think about the role language plays as children verbalise their solution of subtraction stories. Finally, we will discuss mental maths and how to actively involve children in oral activities.

What you will learn in this lesson

- Mathematical language
- Subtraction in context
- Mental Maths

Mathematical Language

As we discussed in Lesson 1 of Part 2, children need many opportunities to be exposed to mathematical language. As we discuss the concept of subtraction, it is necessary to highlight again the idea that mathematical vocabulary helps children to express their thoughts and ideas about mathematical concepts. In subtraction, there are certain key words or phrases that are essential to children's ability to solve problems correctly. For example, the word ‘difference’ signifies the answer that is given when one amount is subtracted from another amount. So, for example, in the number sentence 7 – 2 = 5, we would say that five is the difference. This is a key point of understanding for children if they are to solve word problems accurately. They may come across a word problem such as:

For a child who does not understand the significance of the word ‘difference’, this becomes a challenging question because they cannot work out what they are meant to do. This question is trying to get children to work out how many numbers lie between 6 and 10. They could solve the problem by counting on from 6 to 10, or back from 10 to 6 on a number line.

Children may also solve the problem by working out 10 – 6 = □, or even by recalling their number bonds and saying 6 + □ = 10. It is clear that children will need a great deal of practice with mathematical language in order to successfully identify the key information in the question.

ACTIVITY 1

Brainstorm activities in which you can help children develop their use of the mathematical language related to subtraction.
Commentary

It is a good idea to try and encourage children’s use of mathematical language in fun ways. It is also necessary to remember that you need to ensure you have opportunities to show both language modelled by the teacher, and language being used by the children. Children need to hear examples of how the language should be used correctly, so you could play a game where you call out number sentences, using the different vocabulary, and children need to hold up number cards to show the answer as quickly as possible. You could add an extra element by calling out some addition problems, and children need to recognise that these are not subtraction and therefore not solve the problem. In a similar way to the game Simon Says, anyone who solves the addition problem, would then be out of the game. In addition to this, you also need to hear children using the language themselves as they verbalise their understanding so that you can determine their level of understanding.

Some of the words that you would need to identify include subtract, minus, take away, decreased by, fewer, difference, have left, left over, less than, remain.

Subtraction in Context

The concept of subtraction involves taking a given amount away from another given amount, to find out the difference between the two amounts. You should notice that subtraction ‘undoes’ what addition ‘does’. Because of this relationship between the two operations, they are known as inverse operations.

As we discovered when we looked at addition in context, we should start teaching subtraction by using numbers in a real context. This means that we will use subtraction stories to create a context for the problems that children will solve.

When we get children to solve subtraction problems, we need to encourage them to show all the working that they do using concrete apparatus. We can use a variety of manipulatives such as counters, base ten blocks, ten frames or even 2-D resources such as number lines, drawings or even paper pictures.

As with addition, children need to develop their mathematical fluency in order to solve problems quickly and efficiently. In addition to this, children will work on their Problem-Solving and reasoning skills. The subtraction stories are a helpful here, as children learn to identify the key information in the story, and then to use mathematical language to verbalise their solution strategy.

ACTIVITY 2

Give an example of a subtraction story that would be relevant to the children in your class.

Commentary

Identify items that the children in your class may be interested in, or that may relate to the topic you are covering in class at the moment. Ensure that your subtraction story is simple and clear, so that children can work out what needs to be done to solve the problem.

Initially you might choose to keep to the same sentence structure, such as:

Sandle has 10 marbles.
He loses 3 marbles.
How many marbles does he have left?
As they did with their addition stories, children will need to identify the important information in the subtraction story, and to use this to solve the problem. Once they have become confident in this, you can increase the complexity of the stories.

**Mental Maths**

Mental Maths is usually the first part of our mathematics lessons, and it is where we develop the children’s mathematical fluency. As we get children to practice solving problems in their heads, we are consolidating their number sense and reinforcing many mathematical concepts. Another benefit of Mental Maths is that it improves children’s estimation skills and increases their speed in doing all Maths problems. This is because children learn to look at a problem and estimate what the approximate answer will be. They are then able to determine the best strategy to accurately calculate the answer quickly. Another benefit of improving children’s estimation skills, is that they learn to judge the reasonableness of their answers. This is an important mathematical skill that helps them to check their work.

Mental Maths facilitates a shift from concrete strategies to use more abstract strategies, encouraging children use their knowledge of number facts to perform calculations. In Mental Maths, children also learn to select the best strategy for each problem. They discover that certain strategies take a long time and may not easily allow them to find the correct answer.

Once again, it is necessary for teachers to model the strategies that could be used in Mental Maths, so that children see different methods that could have been used. Teachers also need to encourage children to share their methods so that they have the opportunity to verbalise their understanding. Children often respond differently to the explanation of a strategy given by another child, as opposed to the explanation given by the teacher. It is useful to allow these opportunities as some children may become motivated to try different strategies when they see what their friends have done.

**ACTIVITY 3**

**Game - Fast Maths with cards - 2 more and 2 less**

Watch the video “Game- Fast Maths with cards- 2 more and 2 less” (2:43 minutes) to see how you can play a Mental Maths game with your children.

• What resources do the children have in front of them?
• How do the children solve the problems?
• What do you think will happen if children play this game regularly?
Commentary

In this game, the children have to shuffle their cards, and then turn over the top card. They then need to either add or subtract 2 from the card that is shown. They do not use any manipulatives to help them solve the problem. The idea is that they need to solve the problem quickly in their heads, so that they can continue playing.

It is clear that at least one of the children is a little unsure, and so is hesitant to suggest answers. If children play this type of game regularly, then they become more confident in recalling their number facts, and the pace of the game will speed up. It is essential that children know their number facts well, and that they can recall these quickly and easily without having to count or calculate. If they are able to do this, then they are able to focus on solving problems without trying to work out the number facts first.

Check your understanding: Multiple Choice

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<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) <strong>An understanding of mathematical vocabulary:</strong>&lt;br&gt;A) Will develop in later years of school.&lt;br&gt;B) Helps children understand the question.&lt;br&gt;C) Does not affect children's ability to answer questions.</td>
<td></td>
</tr>
<tr>
<td>2) <strong>Subtraction stories:</strong>&lt;br&gt;A) Make Maths more fun.&lt;br&gt;B) Try to trick children.&lt;br&gt;C) Help children solve problems in context.</td>
<td></td>
</tr>
<tr>
<td>3) <strong>In subtraction stories:</strong>&lt;br&gt;A) Keeping the same sentence structure helps children identify the key information.&lt;br&gt;B) Use situations that are relevant to the children.&lt;br&gt;C) Both A and B.</td>
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<td>4) <strong>In Mental Maths:</strong>&lt;br&gt;A) Children learn to select the best strategy.&lt;br&gt;B) Children need to do exactly what the teacher tells them to do.&lt;br&gt;C) There is no opportunity for discussion.</td>
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REFLECTION

- Think about the Mental Maths sessions in your class.
- How is mathematical language used during these sessions?
- How could you promote the learning of new strategies during these sessions?

Well done you have completed Lesson 6.
In this lesson, we will continue to discuss subtraction. Recognising the patterns of subtraction makes it easier for us to understand and solve problems. We will therefore look at different subtraction strategies and investigate using a number line to solve subtraction problems. Finally, we will consider how learner errors can help us to improve the teaching and learning happening in our classroom.

What you will learn in this lesson
• Subtraction strategies
• Subtraction using a number line
• Learner errors

Subtraction strategies
Initially, children will most likely be using counting strategies to solve problems. Let us look at the problem 8 - 3, and investigate the subtraction counting strategies that children could use:

• Counting back from: When using this strategy children count back from the first number. Children would count from 8..., 7, 6, 5
• Counting back to: Children would count back from 8..., 7, 6, 5, 4, 3 and count the numbers they have counted in order to arrive at the answer.
• Counting up from (complementary addition): Children begin by counting up from the smaller number 3..., 4, 5, 6, 7, 8

As we discussed earlier in Part 2, children need to learn to select the appropriate calculation strategies in order to solve problems efficiently. Teachers need to model these strategies and encourage children to move using counting as their only solution method.

14 - 7
children are likely to say the answer is 7 because they know that 14 is 7 doubled

9 - 5
10 is 5 + 5 and 9 is 1 down from 10 so that means the answer is 4

12 - 4
Take from 12 to get to 10. 10 take away 4 is 6. Then put back the first 2 to get the answer 8
ACTIVITY 1

Look at the subtraction strategies discussed above.

- Write down a subtraction story that you could use with your class.
- How could you help children to identify an appropriate strategy to use to solve the problem?
- What would you do if different children used different strategies to solve the problem?

Commentary

When giving children subtraction word problems, it is important to remember that there are different types of problems to which they should be exposed. Make sure you provide examples of all the different types of problems so that children can develop their understanding and confidence in all of them.

The first type is the combination type. In these problems, we know the total number of items, but we don’t know one of the parts. When solving these problems, part-part-whole diagrams are useful, and so is an understanding of inverse operations. For example, in a combine type problem, children may realise that the number sentence is $10 - 7 = \square$. They could then use the number sentence $7 + \square = 10$.

The second type of problem is the change type. In these types of subtraction problems, we generally have the situation where something leaves or breaks, leaving us with the question How many are left?. However, it is important that we provide a variety of problems where the unknowns are in different positions. For example, children may be asked to find out how many there were in the beginning, or to identify what the change was in the problem. Once again, a knowledge of inverse operations is useful here.

The third type of problem is the comparison type, and these are probably the most tricky for children to understand. In these questions, children are either asked to find out how many more or how many fewer there are. What children need to realise is that, regardless of the words ‘more’ or ‘fewer’, the question is actually wanting to know the amount between two numbers. As we discussed in Lesson 6, ‘difference’ problems are tricky for children, and number lines are helpful when solving them.

Subtraction using a number line

Learners need a conceptual understanding of subtraction – in other words, they need to know why they are solving a problem in a particular way. This can be achieved by learners explaining their actions, rather than solving problems by simply following a rote pattern or ‘recipe’.

Practicing subtraction on a number line helps children to understand that the numbers get smaller as we take quantities away. The number line provides a visual representation that allows them to recognise the patterns in their strategies. For example, in the problem $14 - 6$, children could count backwards for 6 jumps. They would then see where on the number line they landed. This is the simplest way they could try to solve the problem.

![Number line example 1](14 - 6)

If the children have developed their understanding of number a bit more, they may choose to use ten as an anchor number. This means that they will jump 4 places to 10, and then another 2 places to 8.

![Number line example 2](14 - 6)
ACTIVITY 2

Finding the Missing Number

Watch the video “Finding the Missing Number” (5:47 minutes) to see how you can use a number line to introduce new strategies.

• Describe an activity that you could do in your class that will help the children to learn how to find the missing number using a number line.

Commentary

As we have discussed previously, number lines are a useful way to help children to identify patterns. They can see the pattern of the numbers along the number line, as well as the patterns used in different ways of counting. In addition to this, children can also begin to notice the patterns in their solution methods. For example, in the video we see how jumps on the number line are used to help children solve addition and subtraction problems. They are used to this method, and therefore when they are asked to find a missing number using the number line, they already have a foundation of knowledge which can be used to help them.

The children would already be comfortable solving $6 + 4$ on the number line. In the example from the video, they now have to find the missing number using a similar strategy. In doing this, they are reinforcing their understanding of number bonds, and the principles of addition and subtraction. By getting children to practice a variety of problems such as this, they are practicing the number facts shown below:

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Learner Errors

It is important to understand the difference between mistakes and misconceptions in the classroom. Mistakes are incorrect answers due to errors in procedural workings. Learners make such mistakes from time to time, but these are easily corrected. Misconceptions arise from incorrect procedural or conceptual ideas, and they occur repeatedly. They are not easy to correct because learners believe that what they are doing is right when in fact, they are making errors. Teachers need to deal with learners’ misconceptions so that the learners will be able to progress to a more sophisticated mathematical understanding.
The diagram below illustrates the connection between ‘errors’, ‘mistakes’ (or slips) and ‘misconceptions’.

**Mistakes**
- Occasional
- Procedural
- Easily corrected
- Can be self-corrected

**Misconceptions**
- Repeated
- Conceptual or procedural
- More challenging to correct
- Require teacher intervention

**ACTIVITY 3**

Look at the problem:
- What do you think the child's misconception might be?
- What could you do to rectify this misconception?

**Commentary**

In this example, the child has added the digits rather than subtracting them. In addition, he does not seem to understand the concept of place value. We can see that he has added 2 ones and 8 ones to get 10 ones. He also added 7 tens and 3 tens to get 10 tens. He then added 10 tens to 10 ones to get an answer of 1010.

He could use base ten blocks to help him visually recognise the value of the numbers, and to grasp the idea of equal exchange. By using these resources, the child will be able to physically handle the number and construct his own understanding. He could also use a number line to see the movement along the line, and to recognise that when you subtract, the numbers get smaller.
Check your understanding: True or False?

1) Counting strategies are the most effective ways to solve subtraction problems.

2) The three types of subtraction problems include combine, change and comparison.

3) Number lines are useful visual representations of patterns.

4) When children make a mistake, it is always an indication of a serious misunderstanding of a concept.

REFLECTION

• Reflect on your experience of subtraction in the classroom.
• Do children typically find subtraction easy or difficult?
• What do you do when children make mistakes in their work?
• How can you tell if a child has made an error or if they have a misconception?

Well done you have completed Lesson 7.

Answers:
1) False – children should move beyond counting strategies and use calculation strategies instead.
2) True
3) True
4) False - mistakes can be just errors in procedural working.
In this lesson, we will continue discussing subtraction. We will look at the different concrete resources that can be used to help children construct their understanding of subtraction. We will then investigate the use of Flard cards in subtraction problems before talking about base ten blocks.

What you will learn in this lesson

- Subtraction and concrete resources
- Subtraction and Flard cards
- Subtraction and base ten blocks

**Subtraction and concrete resources**

In the same way as they did for addition, children can construct their understanding of subtraction by using concrete resources, before progressing on to pictorial representations. Once children have a sound understanding of the concept of regrouping, they can then move on to working symbolically. It is important that children do not move through the process too quickly, as this will lead to an increased chance of misconceptions being developed.

When children move through the process from concrete manipulatives to pictorial representations until they finally reach the abstract stage, they are able to build on their understanding at each step. This develops a firm foundation resulting in an ability to solve problems with greater confidence and efficiency. Children who do not have this foundation of understanding, tend to try and solve problems through rote learning, or memorisation of a ‘recipe’ or steps in solving a problem. The concern here is that if children forget these steps, they then have no way to solve the problem. They cannot try a different method because they have no understanding on which to draw.

**ACTIVITY 1**

Describe an activity in which you can help children to develop their understanding of subtraction.

- Which concrete resources could you use in your activity to help children grasp the concept?
- How will the use of concrete resources help you assess the children?

**Commentary**

When selecting concrete resources for your activity, think about what you will be trying to teach. If you are working with smaller numbers, then you could use counters and ten frames. In this way children can physically experience the action of taking away counters, and they can see the resulting empty spaces on the ten frames.

If you will be teaching subtraction with larger numbers, then base ten blocks would be useful to represent the exchange process. We will discuss this further in Lesson 9. As the children use the resources, you as the teacher would observe what the children are doing. This will help you to assess the children’s level of understanding. You will be able to see how they handle the resources, and this will give you a chance to recognise their misconceptions. It is important to encourage children to verbalise what they are doing while they solve the problems. The expression of their thought process clearly indicates their ability to reason through a problem.
Subtraction with base ten blocks

When subtracting larger numbers, children can use base ten blocks with a place value mat as concrete manipulatives. Base ten blocks are a particularly useful resource for children to consolidate their understanding of regrouping. As we have mentioned, subtraction is tricky for children to grasp, and the idea of regrouping is challenging for them. They need to be able to see the process, and be able to physically remove a ten from the tens column and exchange it for ten ones. Using the place value mat is helpful as they can then easily see where they need to go to in order to make the exchange.

In the images, we can see that the children realise that they only have 2 ones, but they need to take away 8. They know that they can exchange 1 ten for 10 ones, which then gives them 12 ones in total. They can then take 8 ones away, leaving them with 4 ones.

Equal exchange with money

In maths, there are a number of different topics, concepts and skills that are covered. Very often, teachers approach these separately, teaching each one on its own, before moving on to the next idea. Unfortunately, this approach doesn't allow children to see the inter-related nature of these topics, skills and concepts. As previously mentioned, the more children know about one number, the more they understand about that number's relationship to other numbers. In the same way, a topic such as Money should not be taught in isolation. If the concept of Money is only taught for a few lessons before the teacher moves on to a different topic, then the children are likely to forget what they have learnt. It is a good idea to use what children know about notes and coins to help them develop their understanding of number, as seen in our Smart Counting lessons. Children can use R10 notes to help them count in tens or they can use R2 coins to help them count in twos. These examples will help children see the R10 note as a group of ten, recognising that even though it is only one note, it has the same value as ten R1 coins. This will encourage children to progress from their initial stage of counting all.

ACTIVITY 2

Subtraction Part 1 - using base ten-blocks (no exchange)"

Watch the video “Subtraction Part 1 - using base ten-blocks (no exchange)” (4:13 minutes) to see how base ten blocks can be used to solve subtraction problems.

- Have you used, or seen children using, either of the two methods shown in the beginning of the video?
- Why do you think children find the concept of subtraction difficult?
Commentary

The two strategies shown at the beginning of the video might not be strategies that you have seen children use in the classroom, but it is important for you to know and understand them. As children develop their understanding of subtraction, you may find that one of these methods is used to solve a problem. This is why it is so important to allow children the opportunity to verbalise their methods, so that you can determine what level of understanding they have, and also for other children to hear different methods.

Subtraction is a tricky concept from children, because typically children rely on their knowledge of addition in order to develop their understanding of subtraction. This means that children need a solid foundation of solving a variety of addition problems so that they can confidently begin building on their ideas about subtraction. Unfortunately, this can also mean that some children may make errors in their attempts to solve problems by using learnt addition strategies rather than selecting a method based on their understanding of the problem.

Subtraction and Flard cards

Flard cards are very useful resource in the classroom. The cards help children to recognise the value of each digit in a number, and it also helps them to break down numbers in such a way that it simplifies the calculation process.

When children recognise that 499 is broken down into 400 + 90 + 9, it becomes far easier for them to subtract 10. They just need to take ten away from the ninety, leaving them with 80. Then they can reconstruct the number, saying 400 + 80 + 9 = 489.

Flard cards help children to make the transition from base ten blocks to the abstract written number sentences and column format. It is therefore important that children have a sound understanding of subtraction with base ten blocks, counting in multiples and regrouping before they start trying to move towards more abstract representations.

ACTIVITY 3

Describe how you would use Flard cards with the children in your class.

• How would you get children to prepare for the start of the activity?
• How can you get children to consolidate their understanding of subtraction through a game?

Commentary

It is a good idea to have a large set of Flard cards for whole class demonstrations, so that children can visually follow the discussions. Children can then use their own smaller versions of the Flard cards so that they can be actively involved in the activity. Children should sort the cards into groups according to their place value. The children can lay the cards out in order so that when they are looking for a particular card, it will be easier to find.

You can consolidate children’s understanding of place value by calling out numbers and getting children to construct the number with their Flard cards. You need to ask children to break down numbers (321 into 300 and 20 and 1) as well as build up numbers (200 and 10 and 3 makes 213). You could also ask the children some quick mental calculations such as Show me 10 more than 431 and Show me 5 less than 675.

Children can also play a game where they work in pairs, and they each have to create a number using their Flard cards. Once their numbers have been constructed, they can then add or subtract their numbers. You can get the children to keep a tally record of who solves each problem the fastest.
### Check your understanding: Multiple Choice

1) **Accurate and efficient subtraction involves:**
   - A) Rote learning.
   - B) Following a recipe.
   - C) Building up understanding progressively.

2) **Base ten blocks:**
   - A) Are useful to see the process of exchange.
   - B) Too bulky and awkward to use in a classroom.
   - C) Are only useful for addition problems.

3) **Children find subtraction:**
   - A) Easier than addition.
   - B) Difficult because subtraction is not logical.
   - C) Difficult because children need to rely on their understanding of addition.

4) **Flard cards:**
   - A) Develop children’s understanding of place value.
   - B) Should not be used in subtraction problems.
   - C) Both A and B.

---

### REFLECTION

- Think about your use of concrete resources when teaching subtraction.
- Which resources do you prefer using in the classroom?
- Which resources do the children prefer using to help them?
- What can you do to manage your use of resources more effectively?

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**Well done you have completed Lesson 8.**
In this lesson, we will revisit the idea of pictorial representation. Children will move towards using pictures to solve problems, rather than concrete apparatus, as they consolidate their understanding. It is therefore important for us to have prepared for this progression of learning. We will also discuss using games to help children learn, as well as discussing the role of continuous assessment in Foundation Phase classrooms.

What you will learn in this lesson

- Subtraction and pictures
- Games
- Assessment

Subtraction and pictures

As we discussed with addition, children will eventually begin to move away from concrete resources and begin to use pictorial representations instead. As they consolidate their understanding and become more confident in their ability to solve problems, they will find it easier to draw pictures to solve problems.

Drawing base ten blocks is a more abstract way of solving problems, and children will find it quicker to draw squares, lines and dots to represent hundreds, tens and ones.

Children can use a 100 square to subtract numbers. They will be able to see the patterns that arise from subtracting numbers, and they will learn to use these to help them solve problems (first picture below). Children can also use a number line to solve subtraction problems. They can develop efficient strategies by using the number line, and these will help them to solve problems quickly and accurately (second picture below).
**ACTIVITY 1**

**Subtraction Part 3 - using base-ten blocks (with exchange)**

Watch the video "Subtraction Part 3 - using base-ten blocks (with exchange)" (7:20 minutes) to see how children can solve a subtraction problem that involves equal exchange with their base ten blocks.

- What do you notice the children do in the beginning of the video?
- How do children solve the problem 72 - 45?

**Commentary**

In the beginning of the video, the children initially pack out 72 and 45. The teacher needs to stop the children and remind them that in addition we pack out both numbers, but that in subtraction we only pack out the first number. As we discussed in our previous lesson, this shows us that children rely on their knowledge of addition in order to solve subtraction problems. It is essential that the children are given plenty of time to develop their understanding of the concept of subtraction, so that they are able to choose the appropriate strategy to solve the problem.

In the video, we can see that children practice equal exchange in order to solve the problem. There seem to be few children who are confident in this, and so the teacher models the process for the whole class. It is important to note that children may forget to subtract the tens one they have exchanged and subtracted the ones. Many children tend to think they have then finished the problem, forgetting to subtract the tens. When teaching this strategy, be sure to reinforce this and to look out for children who may develop misconceptions.

**Games**

As we have discussed throughout our lessons, children learn best through play. By playing games, and engaging with resources, children are able to construct their own understanding of concepts.

Most activities can be turned into a game, where children can solve problems in a fun way. When children play, they learn without realising it. They are able to develop their understanding without trying to memorise strategies. They use their knowledge of number and concepts to find ways to get to answers, which helps them to recognise strategies for themselves.

**ACTIVITY 2**

How could you provide opportunities for children to learn their bonds of 10 in a playful manner?

- Describe a game that children could play using number and picture cards from 0 – 10.
Commentary

As we have learnt in Part 1, children learn through play. Children need a variety of opportunities to practise using their mathematical vocabulary in playful situations. The teaching and learning of number bonds is a great way for children to develop their ability to verbalise their understanding, and to participate in game playing. When learning their bonds of ten, children could be given a set of numeral cards and picture cards from zero to ten. Children could work in pairs, turning their cards face down on the table. They could then play a memory game where they try to match cards to make the sum of ten. As they play, they’ll be able to verbalise their understanding of the relationship between numerals and pictures, using the correct mathematical vocabulary to express their thinking.

Assessment

The simple answer to the question, ‘When should assessment take place?’ is that assessment should be ongoing and continuous. Assessment is central to the teaching and learning cycle, but the purpose of the assessment at different points in teaching and learning will be different.

Before beginning teaching on a new topic, you need to determine what the learners already know about what is to be taught. It is also good to give them some insight into how they will benefit from what they are about to learn. Secondly you need to determine the prior knowledge that the learner has already acquired in relation to the topic.

During the lesson, one should do ‘in-process assessment’ (also known as ‘feedback’ or ‘formative assessment’). This is important as it provides information on the learner’s progress on an ongoing basis. It also indicates to teachers and learners if the concepts and skills being taught have or have not been learned and is used in order to plan follow-up teaching and learning.

During this phase, assessment is undertaken at specified times after teaching and learning have taken place. The learners’ achievements are then communicated to them, their parents and the school personnel. This type of assessment can be classified as summative assessment since it provides information about the learning achievements in relation to the curriculum requirements. Sources of obtaining summative assessment results include things such as oral or written tests, and records of structured oral and practical activities. The goal of the assessment process is to support all learners to attain the important mathematical knowledge for the term and grade as it progresses sequentially along the learning trajectories designed into the curriculum.

ACTIVITY 3

• Why do you think that assessment in the Foundation should take the form of continuous assessment?
• Can continuous assessment be used as summative assessment?

Commentary

Continuous assessment is the natural form of assessment in Foundation Phase classrooms, because the children are at the beginning of a journey of lifelong learning. As they journey, they will learn the values, attitudes, skills and knowledge that they need to achieve the goals they set for themselves in life. It is the responsibility of teachers to ensure that they do indeed achieve this learning, as they progress through school.
Formative assessment is a part of continuous assessment – it involves feedback, which can be given if assessment is carried out on a continuous basis, using a variety of methods. Children do not all learn things at the same rate. Factors such as the situation they find themselves in and their individual ability come into play. Children should be given several opportunities to show that they are progressing in the achievement of the learning outcomes. A policy of continuous assessment facilitates the formative use of assessment.

To rely on a final test at the end of a term or year will not allow all children to demonstrate the range of knowledge and skills they have developed, and the stages of their progression. Continuous assessment gives you the opportunity to vary the kind of assessment you are using because you assess the children a number of times and in different ways. The results of all these can be used in the final (summative) assessment of children’s achievement.

### Check your understanding: True or False?

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<tr>
<td>1) Number lines, drawings and 100 squares are good examples of pictorial representations that can be used when teaching subtraction.</td>
<td>True</td>
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<tr>
<td>2) In Maths, games should be limited to specific days, times or topics.</td>
<td>False – Children learn through play, so games should be included as often as possible.</td>
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<tr>
<td>3) Assessment should only take place after a topic or concept has been taught.</td>
<td>False – Assessment should occur before, during and after concepts have been taught.</td>
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<tr>
<td>4) Summative assessment is the only type of assessment that provides accurate information.</td>
<td>False – Continuous assessment provides the most accurate information.</td>
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### REFLECTION

- Think about the way you assess learners in your class.
- Do you follow the guidelines suggested above for assessing before, during and after teaching and learning?
- Discuss at least one idea that you learned about when doing the activities in this session that you can apply in your teaching.

**Well done you have completed Lesson 9.**
In this lesson, we will spend time looking at solving addition and subtraction problems by using the number line. We will expand on our previous discussions of this and consider how we can use the number line to solve problems more efficiently. We will then investigate using inverse operations to check our solutions to addition and subtraction problems. Finally, we will think about how learner errors help us to plan maths lessons in order to address misconceptions.

**What you will learn in this lesson**

- Addition and subtraction using the number line
- Using inverse operations to check
- Learner errors

**Addition and subtraction using the number line**

When children try to subtract numbers in an abstract way, they can make mistakes due to misconceptions or simple errors. If they use a number line, they can begin to think of efficient ways to solve the problem that result in fewer errors.

For example, in the problem 723 – 396, the children need to exchange both ones and tens. This is a skill which can be tricky for children, and so can result in errors. However, if the children use a number line, they can partition numbers in such a way that makes subtracting and bridging the ten easier to manage. Child may decide to count backwards along the line, by counting back in tens first. They could then take away 300. To bridge the ten, children may then break the 6 into 3 and 3.

Some children may find it more efficient to use flexible partitioning, where they break 396 into chunks that make it easier for them to move along the number line. Some children may decide to break 396 into 300 + 70 + 20 + 3 + 3 so that they could bridge the tens in a way that makes sense for them.
**ACTIVITY 1**

Watch the video “Addition and Subtraction using the Number Line Grade 3” (5:44 minutes) to see how children can use a number line to solve addition and subtraction problems.

- What do you notice about the number line used in the video?
- Why is it important to allow the children to select the intervals for the number line?

**Commentary**

In this video, the children use an open number line. This means that, while there are interval demarcations, there are no numbers indicated on the line. This is so that children can fill in their own numbers, and they can decide on the intervals. This is a useful resource, as it gets children to think about counting in multiples, and about choosing the best strategy to solve a problem. If you had the problem 53 + 30, and you decided to go up in ones for your number line intervals, you would need a very long number line to fit everything on. If you use tens as your intervals, then it makes the number line much more practical.

It is necessary to allow children opportunities to count in multiples, where they start from a different number. For example, if you wanted children to count in tens, it is a good idea to get them to start at 7 so they need to say 7, 17, 27, 37, 47, 57 etc. Using a hundred square is a useful resource for this, so that children can see the pattern as they move down the column of numbers.

**Using inverse operations to check**

Inverse means the opposite, and a mathematical operation is the way in which we calculate. The four operations are addition, subtraction, multiplication and division. So, when we refer to an inverse operation in maths, we are talking about an operation that will undo what was done by the previous operation. For example, if we have 5 pencils and we add another 2 pencils, we end up with 7 pencils all together. The inverse of this would be to take 7 pencils and subtract 2 pencils. We would be left with 5 pencils. In this example, addition and subtraction are inverse operations.
Addition and subtraction are inverse operations, so addition ‘undoes’ subtraction and subtraction ‘undoes’ addition. This means that we can use addition and subtraction to check our answers when we’re solving problems. Children can rearrange the numbers in a subtraction number sentence to create two addition number sentences, which can then be used as part of their check after they have solved a problem. For example, if children are given the problem $45 - 34$, children could use base ten blocks or a number line to help them get to an answer of 23. They would write out the number sentence:

$$45 - 34 = 11$$

Children could then rearrange the numbers into:

$$11 + 34 = 45$$ and $$34 + 11 = 45$$

The addition problem could be solved by a quick mental addition of $34 + 10 + 1 = 45$, which tells the children that they have solved the subtraction problem correctly.

**ACTIVITY 2**

Give children a subtraction problem to solve.

- Watch children solve the problem and see their level of understanding.
- Ask children to use addition to check their solution and see whether they are able to use the inverse operation to clarify their understanding.

**Commentary**

The operations have certain rules (or ‘laws’) that you need to know about. The children do not need to know the formal names of these laws, but they do need to be aware of them from quite an early stage. It is important that you as the teacher know the names and the functioning of these laws.

Addition and subtraction are inverse operations, but they do not behave in exactly the same way. Addition is commutative which means that we can add a pair of numbers in any order and still get the same answer. However, subtraction is not commutative. Addition is associative which means that when we add three or more numbers together, we can pair them in any order we choose, without changing the final answer. Subtraction is not associative.

We can use addition and subtraction to check our answers to problems. A part-part-whole diagram is a visual representation of the notion of using inverse operations to check solutions to problems.

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Children can see that: $25 + 12 = 37$  
$12 + 25 = 37$

But they can also see that: $37 - 25 = 12$  
$37 - 12 = 25$
Lesson 10

Learner Errors

According to Ball there are different types of knowledge involved in doing error analysis. Subject knowledge (which Ball and Shulman call content knowledge) enables the teacher to identify an error (is the answer right/wrong?). Identification of an isolated error is just the first step. If a teacher can identify patterns in learners’ work, this shows a deeper understanding of the content from the perspective of the learner. This is what is known as specialised content knowledge. If the teacher can realise what learners were thinking when they made the error, then the teacher is using pedagogic content knowledge. This is a kind of knowledge which is special to teachers since it enables them to understand their learners more deeply and be able to communicate with them more effectively.

The implication of this is that teachers need to explore and diagnose misconceptions. They need to understand where misconceptions and errors come from, and then plan mathematics lessons that will confront children’s misconceptions. If teachers can get children to engage in a dialogue and restructure their thinking, then they are on the correct path to correcting misconceptions.

ACTIVITY 3

Give your children a subtraction problem to solve. For example:

Thandeka has 17 chickens.
Lindiwe has 9 chickens.
What is the difference in the number of chickens?

• Observe how the children solve the problem.
• Can you identify what knowledge the children need to have in order to solve this problem accurately?

Commentary

In this problem, the children have to recognise what the question is actually asking them to do. They need to know the mathematical vocabulary (‘difference’), recognising that they are being asked to find out how many numbers are in between 9 and 17. In order to solve this problem, children need to be able to select the appropriate resources to help them. A number line is a useful resource as it clearly shows the children how many numbers are in between 9 and 17. Children also need to have an understanding of inverse operations, because they could solve this problem by saying $17 - 9 = \square$, or they could say $9 + \square = 17$.

As the teacher, if you noticed your children adding 9 and 17, you would need to then decide what that tells you about their understanding of addition and subtraction. The children may have a limited understanding of mathematical language, which you could focus on in future lessons to correct. Alternatively, children may have a misconception about how to solve subtraction problems. They may be trying to use addition strategies to solve all problems, regardless of the nature of the question, because they have not yet grasped the concept of subtraction. This requires a different direction in future lessons in order for you to address the misconception.
### Check your understanding: Multiple Choice

1) When using a number line:
   - A) Children should count in ones.
   - B) Children need to partition numbers into hundreds, tens and ones only.
   - C) Children can choose to flexibly partition numbers.

2) An open number line:
   - A) Is confusing for children.
   - B) Helps children to count in multiples.
   - C) Should be avoided in Maths lessons.

3) Inverse operations:
   - A) Mean undoing what was done.
   - B) Can be used to check solutions.
   - C) Both A and B.

4) Children’s incorrect answers:
   - A) Help the teacher determine the direction of future lessons.
   - B) Should be ignored as the children are just careless.
   - C) Only important if most children in the class make the same mistake.

**REFLECTION**

- Take some time to think about your own practices in relation to learners who make errors in your mathematics class.
- Reflect on your teaching practices with these questions in mind:
  - How do you address learners’ errors?
  - Could you engage with learners’ errors more effectively? Why or why not?

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**Well done you have completed Lesson 10.**
PART 3: MULTIPLICATIVE OPERATIONS AND FRACTIONS

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In Part 3 you will be introduced to multiplicative operations and fractions which form part of the content covered in the topic of Number, Operations and Relationships. Since, as you have learned about in Part 1 and Part 2, learners learn through play and interaction with other learners and adults, the activities presented in Part 3 show how the content of multiplicative operations and fractions can be taught in a play-based classroom, working interactively and through engaging with errors that arise in the learning process. In this module you will learn about the teaching of multiplication (introduced through repeated grouping activities) and division (introduced through sharing and grouping activities). You will also learn about the teaching of fractions, where it is shown that giving learners opportunities to work with a wide variety of wholes (continuous and discontinuous) is needed. Activities should be set in both in concrete and abstract problem contexts to enable the development of a solid concept of fractions as numbers.
In this lesson we begin our investigation of multiplicative reasoning. Multiplication is not an easy concept for learners to grasp, and you need to be sure that you have a sound understanding of the progression of learning in this topic and will be able to support learners to achieve the necessary conceptual understanding of the topic. We will start by looking at patterns in multiplication, because if learners recognise the patterns in Maths, then they find it easier to learn new concepts. We will also discuss times tables and consider ways to help learners learn these. Finally, we will look at language in Maths so that we can ensure learners use the appropriate vocabulary to verbalise their ideas and understanding.

What you will learn in this lesson

- Patterns in multiplication
- Times tables
- Maths language in multiplication

Patterns in multiplication

As we have discussed in previous lessons, patterns help us to make sense of the world around us. In the same way, if learners are able to identify multiplication patterns, they will find it easier to solve problems involving multiplication accurately and efficiently. You may not explicitly teach these patterns, but rather provide opportunities for learners to discover them for themselves. Ask probing questions that can guide this discovery. This will allow learners to construct their own understanding, rather than trying to remember rules or a recipe to follow.

We mentioned the importance of mathematical vocabulary in the previous section, and part of the language of multiplication includes the concept of multiples. We encounter multiples in both multiplication and division. Learners begin to discover and learn about multiples when they do repeated addition, or drill their multiplication tables. One would expect this to be a commonly known term, and yet it seems that it is not well understood at all. This may be because the term is not used sufficiently, and so we need to be sure to explain the term multiple to the learners and to use it as often as possible.

Some examples of multiples include:

- 2, 4, 6, 8, 10 are multiples of 2
- 7, 14, 21, 28 are multiples of 7
- 60 minutes in an hour are broken up into multiples of 5 on an analogue clock face

All numbers have an infinite number of multiples, while some pairs of numbers share certain multiples. To find common multiples, we simply write out some of the multiples of the given numbers and then look for those which are common to both. For example, numbers that are multiples of both 2 and 3 are called common multiples of 2 and 3.

Multiples of 2

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

Multiples of 3

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

6, 12, 18, 24 and 30 are common multiples and 2 and 3.
ACTIVITY 1

Watch the video “More multiplication patterns” (2:59 minutes), to see how the teacher uses multiplication patterns.

• Why do you think the teacher had blue and green marbles in the bags?
• What patterns do you notice in the video?

Commentary

In this video, we notice that the teacher has used blue and green marbles in the bags. She first asks the learners how many marbles there are altogether. The learners can see that there are 5 marbles in each bag so they can quickly work out that there are 20 marbles altogether. The learners do this by using repeated addition (5 + 5 + 5 + 5 = 20) which we will discuss more in Lesson 2. The teacher then looks at the number of green and blue marbles, which encourages learners to break down 5 into 3 + 2 for each bag. Learners can therefore see a pattern in that all the bags have 3 + 2 = 5 marbles. They can also work out the number of marbles by counting in 2s for the green marbles, 3s for the blue marbles and 5s for the total number of marbles. So, by using the blue and green marbles, there are three different calculations that can be worked out, even though there are only four bags of marbles in total.

Multiples of 2 pattern

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Multiples of 5 pattern

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Times tables

It is essential that learners know their times tables well, as so that they are able to recognise the relationships based on them. This will help learners to develop their confidence and accuracy in solving mathematical problems. For example, learners know that 9 x 7 = 63 because 10 x 7 = 70 and 7 less is 63. Based on their knowledge and understanding of patterns and relationships, learners are able to solve problems like these efficiently.
As part of identifying patterns and relationships, learners need to see the link between counting in multiples and “times tables”. Encourage learners to compare parts of the tables. So, for example, learners can look at \(8 \times 4 = 32\) and \(9 \times 4 = 36\). You can ask them what they notice, and what they think the next number in the sequence will be. Learners need to recognize that \(9 \times 4\) is one group of 4 more than \(8 \times 4\).

Learners therefore need a lot of practice with their multiplication tables. However, it is important that this practice should take place after the concept of multiplication has been thoroughly understood by the learners. Remember also that learners learn best through play, so the practicing of times tables should be made exciting and interesting for the learners by using games and other classroom activities.

**ACTIVITY 2**

How could you teach the four times table to learners?

- What can you do to make sure that learners construct their understanding rather than learning the times tables by rote?

**Commentary**

When learning their times tables, learners must build up each table so that they can see and understand where it comes from.

When learning the 4 times tables table, you could show the learners a toy car. Point out that you are holding one car, and that one car has four wheels. You can ‘park’ the car and write on the board \(1 \times 4 = 4\), saying as you write “1 car with 4 wheels has 4 wheels in total”. You can then drive up another car, parking it next to the first car. You can write \(2 \times 4 = 8\), saying “2 cars with 4 wheels each have 8 wheels altogether”.

You can carry on this way until you get to “12 cars with 4 wheels each have 48 wheels in total”. You can then go back to the beginning and discuss with the learners what they think will happen if we have zero cars. Let the learners see if they can write the number sentence by following the pattern that they have developed. The learners will see that if they have 0 cars with 4 wheels, then there will be 0 wheels in front of them.

**Maths language in multiplication**

The language of multiplication should be taught and used regularly in your lessons. Some of the terms associated with multiplication include multiply, times, groups of, sets of and multiples. Learners need to be comfortable using the correct terms as they verbalise their thought processes. It is important that learners know exactly what a multiplication problem is asking them to do. This means that, when they see the number sentence \(4 \times 5 = \square\), they understand what the multiplication symbol means. Learners need to be able to recognise that the number sentence is actually saying “4 groups of 5”, which will help them to solve the problem. Make sure that learners know that it is not correct to say “times this number by 7”. The correct language is “multiply this number by 7”. We do however speak about “7 times 5” which means “multiply 5 by 7”.

As with addition and subtraction, when we introduce multiplication to our learners, we should use number stories that will lead to simple multiples. Make up stories yourself and call on the learners to make up some of their own too. If you allow them to make up their own questions from this early stage, they will develop their independence and ability to think creatively about mathematical situations.
ACTIVITY 3
Make up a story that would lead to a number sentence involving multiplication.

Commentary
As we discussed with addition and subtraction, you need to select items that interest the learners in your class. Then you can come up with problems, using simple sentences, that follow the same basic structure.

Encourage the learners to create their own multiplication stories, thinking about how to use the mathematical language that they have learnt. Make sure that you have modelled the appropriate use of the language so that the learners have a good example to follow.

Check your understanding: Multiple choice

1) Which sequence of numbers are multiples of 7?
   A) 2, 4, 6, 8, 10
   B) 7, 14, 21, 28, 35
   C) 1, 7, 17, 27, 37

2) I know that 4 x 8 = 32 because:
   A) I know that 4 x 10 = 40 and I can count backwards.
   B) I know that 4 x 5 = 20 and I can count on in 4s.
   C) Both A and B

3) The language of multiplication includes:
   A) Groups of, add, put together
   B) Groups of, sets of, times
   C) Groups of, sets of, add

4) When teaching multiplication, it is helpful to:
   A) Use problems in context
   B) Make learners learn algorithms off by heart
   C) Practice first, understand later

REFLECTION
• Reflect on your experience of learning times tables.
• Think about you felt when you were tested on your times tables.
• Write a personal goal in relation to the way you plan to approach times tables in your class.

Well done you have completed Lesson 1.
In this lesson, we will look into multiplication a bit more. We will start by considering how learners use concrete resources when solving multiplication problems. We will then move on to thinking about repeated addition, as this is how learners start solving multiplication problems, before talking about using arrays. This is the progression of learning that learners will follow, as they move from concrete apparatus to pictorial representations. We discussed the progression of learning when we learnt about addition and subtraction, and this progression is continued for multiplication and division.

What you will learn in this lesson

- Multiplication using resources
- Repeated Addition
- Multiplication using an array

**Multiplication using resources**

As we discussed when we looked at addition and subtraction, learners need to use concrete resources to help them visualise the process of multiplication. You can provide opportunities for learners to use counters or cubes, base ten blocks or a number line. Learners can physically arrange the resources into equal groups by using ten frames or circles as we saw in the video in Lesson 1.

Use multiplication stories to provide a variety of opportunities for learners to make equal groups. Let them talk amongst themselves about how they solve the problems. In order to think multiplicatively, learners must realise that they need to count equal groups. By physically grouping items into equal sets, they will become more efficient in their ability to reason and solve problems.

Once learners have used concrete resources to create a solid foundation of understanding, they can use arrays as a pictorial representation of multiplication. We will discuss arrays in more detail later in this lesson. Encourage learners to physically group concrete objects, or to draw circles around dots on an array. When counting items or dots, you should encourage learners to use skip counting so that that they solve problems more efficiently.

**ACTIVITY 1**

- Discuss the resources used to teach multiplication by teachers at your school.
- Which resources are most commonly used?
- Why are these the preferred resources?
- Which resources are not seen as valuable or helpful in the classroom?
- Why are these resources not highly regarded?
Commentary

It is a good idea to discuss teaching strategies and resources with your colleagues. Teachers all have their own teaching methods and styles, and we should generate conversations with other teachers to learn from their experiences. Just as we encourage learners to verbalise their solution strategies, so should teachers verbalise their teaching methods. By talking to others about what they are doing in the classroom, teachers will learn new ways of approaching challenging concepts with the learners. They will also discover which strategies are likely to be successful, and where potential problems may arise.

Repeated Addition

As we discussed in Lesson 1, when learners are given a multiplication problem to solve, they need to work out what they are being asked to do. Learners develop their understanding of multiplication through solving word problems that incorporate the appropriate language. For example, the problem might be:

Learners could then use either concrete apparatus or drawings to help them solve the problem.

It is important to encourage learners to verbalise their solution, and many learners may say “2 and 2 and 2 and 2 and 2 and 2 and 2 and 2 and 2 and 2 and 2 makes 22 wheels”. If they verbalise the solution in this way, you should then encourage learners to understand that they could use a multiple rather than repeated addition. They could ask themselves “How many groups of wheels are there?” Help learners to realise that there are “11 sets of 2 wheels”. This will help them to move away from repeated addition towards multiplication.

Once learners have developed their understanding of repeated addition, showing an ability to explain what they are doing to solve the problems, you can start encouraging them to calculate more efficiently. You can give learners a word problem where the repeated addition would be tedious. For example, “There are 12 tricycles. How many wheels altogether?” This will take learners a long time to draw or write out as a repeated addition problem.

You could say to the learners “This is so much to write down. Can you think of a shorter way of writing this out?” You can encourage learners to recognize that there are 12 groups of 3 which can be written as $12 \times 3 = 36$. By doing this, learners will begin to think more efficient ways of solving multiplication problems. Using multiples is efficient and appropriate. Moving away from repeated addition is essential. The next activity would be appropriate at the Grade 3 level.

ACTIVITY 2

Solve the multiplication problem $63 \times 5$ without using a calculator.

• How did you solve the problem?
• Could you have found an easier way?
Commentary

As adults with an existing knowledge of multiplication, we might solve the problem 63 x 5 by using the column format (first calculation on the left). Learners who are not confident with the column format (or have not learned it) might choose to use repeated addition. Some alternatives are shown below.

It can be tricky to avoid errors when doing repeated addition. There are often a lot of numbers to add, and it can be easy to make mistakes. This leads learners to realise that multiplication is an easier solution method. There are even other ways that this calculation could be done – try to find a few!

Multiplication using an array

While it is a good idea to introduce times tables using concrete resources (as we discussed in Lesson 1), this is not always practical in terms of managing the resources in the classroom. An array is a great alternative to concrete resources, as you can place a cover over parts of the array and show any multiplication fact you want from 1x1 up to 10x10. In an array, we arrange dots (or other shapes) in columns and rows. We can then use the array to find the product of any multiplication question. For example, to multiply 3 x 5, we will create the following multiplication array.

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In this array, we can see that there are 3 rows and 5 columns. We can think of this as 5 + 5 + 5 or as 3 x 5, and we can work out that there are 15 dots altogether. It is useful to write out your number sentences the same way each time to avoid confusion. In other words, the first number in the number sentence could be the number of rows, while the second number represents the number of columns. Ultimately learners realise that 3 x 5 is the same as 5 x 3.
Arrays provide a visual representation of repeated addition or multiplication. A benefit of using arrays is that they help learners to see that the order of the numbers doesn’t matter in multiplication. Learners can turn the arrays around to see this clearly. As shown below learners will be able to see that $2 \times 6$ is the same amount as $6 \times 2$.

![Array Diagram](image)

**ACTIVITY 3**

**Multiplication using array diagrams**

Watch the video “Multiplication using array diagrams” (2:57 minutes), to see how the teacher addresses the idea of an array.

- Why do you think the teacher uses a ten frame in the beginning of the video?
- How could you get learners to develop their understanding of multiplication by using arrays?

**Commentary**

In the video, the teacher uses a ten frame as an array so that the learners can easily rotate it to see that $2 \times 5$ is the same as $5 \times 2$. This is the commutative property of multiplication which we will discuss further in Lesson 3.

In order to develop learners’ understanding of multiplication, you could get them to show equal groups using an array. For example, you could ask them to use their array to show 4 groups of 3. The learners could then identify the number sentence $4 \times 3 = 12$. You could also get learners to break up the array in different ways. For example, a $4 \times 5$ array can be broken up into a $3 \times 5$ piece and a $1 \times 5$ piece. This will show learners that $4 \times 5$ is equal to $(3 \times 5) + (1 \times 5)$.

We know that learners move through three stages in terms of their understanding: concrete, representational, and finally abstract. You could use ten frames as a concrete array before moving on to representing the array in a drawing. Lastly, learners can solve multiplication problems in an abstract way, using only symbols. Remember that lots of practice in the concrete and pictorial stages to work efficiently and effectively in the abstract stage.
Check your understanding: True or false?

1. Concrete resources are a great way for learners to learn about equal groups.

2. Repeated addition is a quick and efficient method to solve problems.

3. An array is a visual representation of the laws of multiplication.

4. Based on their experience in adding and subtracting, learners can skip the concrete stage of learning in multiplication.

REFLECTION

• Reflect on your experience of using arrays in the classroom.
• Are you comfortable using an array to solve problems?
• Write a personal goal in relation to developing learners’ confidence in using arrays.

Well done you have completed Lesson 2.
In this lesson, we will investigate the laws of multiplication. It is necessary for you to understand the rules that multiplication follows so that you can teach these processes to the learners. The learners do not need to know the names of the laws, but they do need to recognise certain patterns that will help them to solve problems more efficiently. We will look at using base ten blocks in multiplication, considering how the concrete resources help learners to recognise and work with the laws of multiplication (even if they don’t know their names). Finally, we will discuss assessment, thinking about how to promote successful learning experiences for the learners.

What you will learn in this lesson

- The laws of multiplication
- Multiplication using base ten blocks
- Assessment

The laws of multiplication

Like addition and subtraction, multiplication has certain laws of operation. It is important for you to know these laws and to understand what they mean. The first law is that multiplication is **commutative**: This means that we can multiply a pair of numbers in either order, without changing the answer.

For example, \(7 \times 9 = 9 \times 7 = 63\).

One of the strategies that we can teach our learners is to look out for “easier” ways of ordering in questions that involve multiplication of more than one pair of numbers. Another law is that multiplication is **associative**. This means that if we have to multiply a string of three or more numbers, we can do so by pairing them in any order that we choose.

For example, \(3 \times (2 \times 6) = (3 \times 2) \times 6\).

A third law of multiplication is that it is **distributive** over addition and subtraction. This means that we can rearrange numbers in a number sentence to help us solve the problem in an easier way. For example, in the number sentence \(2 \times (3 + 4) = \square\), we can solve the problem by adding 3 + 4 and then multiplying by 2 to get an answer of 14. However, we could also multiply 3 by 2 to get 6, and then multiply 4 by 2 get 8. We could then add 6 + 8 to get an answer of 14. We could do the same thing with a number sentence that has subtraction instead. For example, in the number sentence \(3 \times (10 - 7)\), we could solve the problem by working out 10 - 7 = 3, and then multiplying 3 by 3 to get 9. Alternatively, we could multiply 10 by 3 to get 30, and then multiply 7 by 3 to get 21. We could then take 21 away from 30, leaving us with an answer of 9.

Multiplication has an **identity element**. This is the number which, when we multiply by, it has no effect on the multiplicand. The identity element of multiplication is the number 1, since when you multiply a number by 1 it does not change.

For example, \(1 \times 8 = 8 \times 1 = 8\)

Finally, it is also necessary to know that when a number is multiplied by zero, the answer will always be 0.
ACTIVITY 1
Test the laws of multiplication by working out the following examples:

- Calculate the following by pairing in different ways:
  \[2 \times 17 \times 50 = \square\]

- Was there an order that was easier for you to do? If so, which one and why was it easier?

- Calculate the following:
  \[13 \times 15 + 13 \times 5 = \square\]
  \[13 \times 20 = \square\]

- What did you notice about the calculations?

Commentary
When you solve the problem \(2 \times 17 \times 50\), you could work it out in the following ways:

\[
2 \times 17 = 34
\]
\[
34 \times 50 = 1700
\]

This is actually quite tricky to work out, as \(34 \times 50\) is not the easiest calculation to work out in your head. However, if you chose to solve the problem like this:

\[
50 \times 2 = 100
\]
\[
17 \times 100 = 1700
\]

It is much easier to solve the problem mentally when you rearrange the numbers in a different order. \(50 \times 2\) is a doubling fact that can be recalled quickly, and \(17 \times 100\) is a simple calculation when you know about multiplying with tens and hundreds. These two methods of solving the problem show the associative law of multiplication.

When you solve the problem \(13 \times 15 + 13 \times 5\), you need to work out \(13 \times 15\) (which equals 195) first, and then \(13 \times 5\) (which equals 65) before adding the two amounts together. So, you would add 195 + 65 to get an answer of 260.

To do this in another way, you could work out \(13 \times 20\) - which you’ll realise comes to the same answer of 260.

In this activity, you should have realised that the two calculations came to the same answer, despite the fact that you solved them in different ways. These calculations show the distributive law of multiplication. You need to give your learners opportunities to discover more efficient alternative ways of getting to solutions by combining numbers in different ways.

Multiplication using base ten blocks
When solving multiplication problems, it is useful to pack out base ten blocks as this can show the laws demonstrated in a concrete way. The base ten blocks are a physical representation of how we can rearrange quantities to help us solve problems more easily. Learners don’t need to learn the laws of multiplication by name, but they will understand the principles of these laws by seeing the blocks laid out and moved around to create new groups.

For example, using base ten blocks to solve a problem where a double-digit number is multiplied by a single digit number works well to show the distributive law. In solving this problem, the learners could attempt a variety of strategies, but they may find it easier to break up the double-digit quantity into tens and ones. They can group the tens together, making it easy to see how many there are altogether. They can then do the same with the ones. A physical representation of the tens and ones using base ten blocks will make it easier for the learners to multiply through a process of repeated addition. As you can see, this is exactly what we do in the distributive law, but it is far easier to understand it when we see it in a concrete form as opposed to an abstract number sentence.
ACTIVITY 2

Watch the video “Multiplicative reasoning 4 – using base ten blocks” (4:35 minutes), to see how the learners use base ten blocks to practice the distributive law.

• How do the learners pack out the base ten blocks?
• What do the learners need to do to help them solve the problem?
• How can you write out the process as number sentences?

Commentary

In the video, the learners are asked to solve the problem 4 x 12. They need to pack out the base ten blocks into tens and ones, recognising that 12 is made up of 10 + 2. They then need to group all the tens together and all the ones together (first image below). By grouping the tens together and the ones together, the learners can see that they have 4 groups of 10, and 4 groups of 2. This helps them to break up 4 x 12 into 4 x 10 + 4 x 2, which simplifies the calculation process for them (second image below).

Assessment

When assessing learners’ learning of multiplication, you need to provide a number of opportunities for them to show you what they understand about the process of multiplying. You will then be able to collect evidence through observation and engagement with the learners. Once you have gathered the information you need from working with the learners, you can evaluate the evidence and make decisions about the learners' levels of understanding. Following this, it is important for you to record your findings, so that you can refer back to these at a different time. As we have discussed previously, it is essential that you use information gathered from assessing learners to plan for future lessons, and to redirect the teaching and learning process where necessary.

It is difficult to accurately assess too many learners at one time. This is why some teachers revert to summative assessment, as it seems easier to just mark a written task and see how many problems the learners were able to get correct. However, summative assessment does not take into consideration the learners' progression of learning or their context. This means that, sometimes summative assessment might not be as accurate or informative as you would like.
Therefore, when setting up formal assessment tasks with your learners, try to work with a group of learners at a time so that you have the opportunity to interact with them. It may take a few days for you to complete your assessments, but you will end up with a far deeper understanding of the learners’ knowledge and skills if you have been able to ask them questions and to listen to their justifications.

**ACTIVITY 3**

Plan and teach an activity where you can assess your learners' understanding of multiplication.

- Design an observation sheet that will focus your interaction with the learners.
- Think about how you will record your observations of the learners.
- How will your observations direct your follow-up lessons?

**Commentary**

Observations about the learners in your class should be factual, accurate and objective (exactly what you see and hear). It is important that you do not allow your own opinions, biases or assumptions to influence what you observe, as this may affect your observation in a negative way. To make sure that observations are factual, accurate and objective consider the following ideas:

Write down the facts. What is the learner doing? What are they saying?

- “Julie is sitting at the table, doing a puzzle. She has placed five pieces together. She is holding one piece and trying to find where to place it.”

Think about whether someone else observing would describe the learner’s behaviour in the same way. For example, two observers might interpret the behaviour differently:

- “Julie can’t finish the puzzle. The remaining pieces are just lying on the table.”
- “Julie has sorted the remaining puzzle pieces according to their colour so that she can join the pieces together that are the same colour.”

Do not make assumptions. Do not write what you think is happening about what the learner can or cannot do, or how you think the learner is feeling. You may miss things that do not fit with your opinion.

- “Julie can’t do the puzzle. She is confused and frustrated and angry and is trying to force the piece to fit.”

Describe what you see and hear in as much detail as possible. This will help you to identify the learner’s achievements and needs as accurately as possible.

- “Anathi is matching number symbols and number names. He puts all the symbols 1 to 8 in order on the table. Then he puts the number names from one to six next to the matching symbol. He holds the number cards seven and eight in his hand and starts to cry, saying ‘I don’t want to do this.’”

Be aware of your own biases. You may have different expectations for girls and boys, or you may assume that a learner with barriers to learning is at a disadvantage. It is important not to jump to conclusions and give each learner the same consideration.

Teachers can also create an observation checklist. This is used to indicate whether a learner is competent, partially competent or not yet competent with a particular concept/skill.

Checklists provide columns into which you can enter information about the achievement (fully/partially/not at all) in relation to given criteria. The first exemplar checklist can be used after teaching a specific topic or at the end of a series of lessons to record more than one topic.
### Mathematics: GRADE 1: TERM 3: Checklist

<table>
<thead>
<tr>
<th>Numbers, Operations &amp; Relationships</th>
<th>Patterns, Functions &amp; Algebra</th>
<th>Space &amp; Shape</th>
<th>Measurement</th>
<th>Data handling</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ - achieved</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>▶ - not yet</td>
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<tr>
<td>● - almost</td>
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</tbody>
</table>

#### Observations:
- Estimates and counts out objects reliably to 40 by using the strategy of grouping.
- Counts forwards and backwards in Ones from any number between 0 and 80.
- Counts forwards in multiples of 10s, 2s and 5s between 0 & 80.
- Recognises, identifies, reads and writes number names 1 to 10.
- Reads number symbols 1 to 80.
- Recognises the place value of numbers 11 to 15.
- Decomposes two-digit numbers into ten ones.
- Copies, extends and describes simple number sequences in 1s, 10s, 5s, 2s to at least 80.
- Recognises and draws line of symmetry in 2-D geometrical and non-geometrical shapes.
- Estimates, measures, compares, orders and records length using non-standard measures.
- Uses language to talk about the comparison.
- Answers questions about data in pictograph.

#### Criteria checklist: correct/incorrect/almost

**Mark: 7**

- Able to match counters to objects ✓ ▶ ●
- Able to sort counters onto a five frame ✓ ▶ ●
- Able to count a number of objects up to 5 ✓ ▶ ●
- Able to compare numbers to say which one is more (greater) than another ✓ ▶ ●
- Able to compare numbers to say which one is less (smaller) than another ✓ ▶ ●
- Able to recognise the number symbols 0 to 5 ✓ ▶ ●
- Able to write the number symbols 0 to 5 ✓ ▶ ●

This rubric is designed to make a record of levels of ability in a Grade 3 class in relation to levels of understanding of the topic of symmetry.

### Space and shape: Assess the learners’ ability to recognise and work with symmetry.

<table>
<thead>
<tr>
<th>Level</th>
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<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
</tr>
</tbody>
</table>

### Observations:
- Able to identify the shape but unable to recognise when a shape is symmetrical 1
- Able to recognise when a shape is symmetrical but cannot show the line of symmetry 2
- Able to recognise when a shape is symmetrical and show one line of symmetry 3
- Able to recognise when a shape is symmetrical and can show more than one line of symmetry 4
- Able to draw a symmetrical shape with one line of symmetry 5
- Able to draw a symmetrical shape with more than one line of symmetry 6
- Able to draw a symmetrical shape or pattern and describe symmetry in patterns where more than one symmetrical shape is present 7

---

*SBA (2019), Grade 1 Checklist.*

The second exemplar checklist is designed as an observation tool for the assessment of learners’ progress in the learning of numbers (0-5).
You can design your own checklists or rubrics to record observations. Take time to think carefully about the criteria you develop for the instruments, so they enable you to distinguish between levels of understanding of the topic under observation.

### Check your understanding: Multiple Choice

1) Which number sentence below proves the commutative law of multiplication?
   - A) $9 \times 2 \times 3 = 9 \times 6$
   - B) $9 \times 3 = 3 \times 9$
   - C) $9 \times (3 + 2) = 9 \times 3 + 9 \times 2$

2) Which number sentence below proves the associative law of multiplication?
   - A) $4 \times 5 \times 2 = 4 \times 10$
   - B) $4 \times 7 = 7 \times 4$
   - C) $4 \times (3 + 5) = 4 \times 3 + 4 \times 5$

3) Observations for assessment should be:
   - A) Brief
   - B) Subjective
   - C) Factual

4) Teacher bias is:
   - A) A myth
   - B) A gut feeling based on experience
   - C) An assumption to be avoided

### REFLECTION

- Think about how you assess learners in your class.
- Do you make assumptions about what you think they know?
- Where do these assumptions come from?
- What can you do to prevent these assumptions influencing your assessments?

**Well done you have completed Lesson 3.**
In this lesson, we will discuss the concepts of doubling and halving. These processes form an important part of learners’ number fact knowledge, and a sound understanding of doubling and halving will help them to solve problems accurately and efficiently. We also discuss the mathematical language of division, as we have done for addition, subtraction and multiplication. Finally, we investigate patterns in division, and how these help learners to simplify the way they solve division problems.

What you will learn in this lesson

- Halving and doubling
- Maths language in division
- Patterns in division

Halving and doubling

When learners understand halving and doubling, they are able to solve certain problems mentally, using their knowledge of number facts. It is therefore essential that you find practical ways to teach the concepts, giving learners multiple opportunities to work with concrete apparatus in playful ways.

An array allows learners to see the laws of multiplication and division concretely. They can physically handle the apparatus. For example, if learners are given the problem 4 x 5, they can create an array of 4 rows and 5 columns. They can then arrange the counters in different ways to help them solve the problem. If the learners rearrange the rows into 2 rows of five, they can see the problem has changed to 2 x 10. The two displays of marbles on the next page illustrate these two different ways in which the arrays can be arranged.
This is interesting because if you look at the two number sentences found here, you will notice a specific pattern. If you halve the first number in the number sentence, and double the second number, you will get the same answer as you would have from the original number sentence.

\[
4 \times 5 = 20 \\
\text{halve} \quad \text{double}
\]

\[
2 \times 10 = 20
\]

This is a useful strategy for solving multiplication problems, but it is important for learners to understand the strategy. They need to realise that they should only use halving and doubling to solve a problem, if it is the most efficient way. For example, to try and solve \(17 \times 15\) by halving and doubling you would do the following:

\[
17 \times 15 = 20 \\
\text{halve} \quad \text{double}
\]

\[
8,5 \times 30 = 20
\]

This is not an easier way to solve the problem. There are other strategies that could be used more easily.

**ACTIVITY 1**

**Halving**

Watch the video "Halving" (3:26 minutes), to see how the teacher uses base ten blocks to help the learners halve.

- How would you introduce halving?
- Do you think the base ten blocks are helpful in getting learners to develop their understanding?
- Would you use a similar method to teach doubling?
Commentary

In this video, the teacher demonstrates halving the number 68 using base ten blocks. She packs out 6 tens and 8 ones so the learners can clearly see how the number is constructed. This then makes it easier for the learners to halve, as they can halve the tens and the ones separately. If they have 6 tens, they are able to easily halve that and say that there will be 3 tens. They can also halve 8 ones, recognising that they will be left with 4 ones. When they add 3 tens and 4 ones, they will see that half of 68 is 34. This is one way to do halving. Try to think about other ways and other contexts that lend themselves to solution of problems using doubling and halving.

Maths language in division

As with all of the other operations, learners need to practice using the maths terminology to express their understanding of division. Learners need to be able to use the language to talk about the way that they solve problems. In the beginning, the concept of division can be explored without the learners even realising that they are dividing. By giving them real life problems, the learners can develop their understanding of the concept in a practical way. For example, the teacher could give learners the following problem:

I have 15 hats.
The hats must go on 3 shelves.
How many hats will go on each shelf?

The learners can use concrete resources to solve the problem, and they need to be able to verbalise their thinking. They are likely to use social language initially, so it is important for teachers to model the appropriate language to help learners develop their conceptual understanding. As learners copy the teacher, and practise using the language, they can share their methods and learn from each other. Once they are comfortable doing this, the teacher can then introduce the language of division. She can teach the learners the words divide, share, group and remainder, giving them a variety of word problems to demonstrate the meaning of these words. As learners develop their confidence in their ability to solve division problems, teachers can then introduce the symbols of division.

ACTIVITY 2

- Write about some activities that will encourage the use of the new vocabulary of division - division, divide, group, share, grouping and sharing.
- When would you do these activities?

Commentary

Learners will develop a better understanding of the appropriate mathematical terminology if they are given many opportunities to practice using the language in context. You need to give them many different grouping and sharing stories that lead to problems they will solve. The word divide or division only is used from Grade 3 onwards. However, it can be challenging for teachers to find time to focus on this during their planned maths lessons. We must remember that we do not need to only use division stories during scheduled division tasks. We can give learners Mental Maths problems that encourage them to use the language of division, even if we are focusing on a different topic in the actual Maths lesson. We could also prepare ‘Pick up’ activities which are available for learners to pick up and work on independently when they have completed set tasks. In this way, learners will be able to practice their division stories regularly and maintain their understanding of the terminology and the problems.
Patterns in division

Knowledge of place value and number properties such as divisibility can help learners to carry out a range of calculations mentally. When learners know their multiplication and division facts, they are able to use this knowledge to solve many problems in their heads simply by understanding the divisibility rules. Divisibility rules are patterns of division (some are simple, some more complex) that allow us to check whether certain numbers are divisible by other numbers (with no remainder) without actually dividing. Divisibility rules therefore enable us to check quickly whether a number is a factor of another number.

**Divisibility rules**

- **2** Even numbers are divisible by 2
- **3** The sum of the digits must be divisible by 3
- **5** Numbers that end in 5 or 0 are divisible by 5
- **6** The number must be divisible by 2 and 3
- **7** The last 3 numbers must be divisible by 8
- **9** The sum of the digits must be divisible by 9
- **10** Numbers that end in 0 are divisible by 10

**ACTIVITY 3**

Solve the problem and then answer the questions below:

*I have 20 biscuits to pack with 5 biscuits in each box. How many boxes will I need?*

- Will this divide without leaving a remainder?
- How did you decide?
- What other numbers will divide into 20 without leaving a remainder?
- How did you decide?

**Commentary**

When solving a problem like this, learners need to make sure that they understand the key information. Encourage learners to look carefully at the number facts in the problem, and to verbalise in their own words what they think they need to do to solve the problem. Watch the learners as they solve the problem and note whether they use their knowledge of multiplication facts to help them. Do learners remember their 4 and 5 times tables? They could use the number fact $4 \times 5 = 20$ to help them work out that $20 \div 5 = 4$. Talk to the learners about their responses and their reasons. Give them opportunities to explain ideas such as “2 will work because 2 tens is 20 and 20 is an even number so it will divide equally by 2”.
Check your understanding: True or false?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1)</td>
<td>Halving and doubling can be used as an efficient solution strategy for multiplication problems.</td>
</tr>
<tr>
<td>2)</td>
<td>Maths language can only be developed in scheduled Mathematics lessons.</td>
</tr>
<tr>
<td>3)</td>
<td>Patterns only fall into the curriculum topic, Patterns, functions and algebra.</td>
</tr>
<tr>
<td>4)</td>
<td>Learners should have a sound understanding of number facts to help them solve problems.</td>
</tr>
</tbody>
</table>

REFLECTION

- Reflect on your experience of patterns in Mathematics.
- Do you use your recognition of patterns in Maths to help you solve problems?
- How can you encourage learners to identify and use patterns in their own Maths lessons?

Well done you have completed Lesson 4.
In this lesson, we will look at the laws of division and what learners need to know about dividing. As they develop their understanding of division, learners need to recognise the difference between sharing and grouping. We will investigate these differences and discuss examples of problems for both sharing and grouping.

What you will learn in this lesson

• The laws of division
• Grouping
• Sharing

The laws of division

We will now look at the laws of division – and what we find is the laws that applied to multiplication do not apply to division. Unlike multiplication, division is not commutative: This means that we cannot change the order of the numbers in a division number sentence without changing the answer. This can be seen in the following pair of number sentences:

\[
35 \div 7 = 5 \text{ and } 35 \div 5 = 7
\]

Division is also not associative. This means that we cannot pair numbers in different orders when working with three or more numbers. This is shown in the following pair of examples:

\[
(36 \div 6) \div 2 = 6 \div 2 = 3 \text{ and } 36 \div (6 \div 2) = 36 \div 3 = 12
\]

Unlike multiplication, division is not distributive over addition and subtraction. This means that we cannot rearrange numbers in a number sentence to help us solve the problem in a different way. For example, in the number sentence \(35 \div (2 + 5) = \square\), we solve the problem by adding 2 + 5 and then dividing 35 by 7 to get an answer of 5. However, if we were to try solve the problem by saying \(35 \div 2 + 35 \div 5 = \square\) we would end up with \(17.5 + 7 = 24.5\) which is not the correct answer.

When you divide 0 by another number the answer is always 0. For example: \(0 \div 2 = 0\). This means 0 sweets shared equally amongst 2 learners, resulting in each learner getting 0 sweets. When you divide by 1, the answer is the same as the number you were dividing. \(2 \div 1 = 2\). For example, if two sweets are divided by one learner, then the one learner would get both sweets.

Division by 0 is not defined. This is often misunderstood in mathematics and needs to be dealt with carefully so that learners understand it better and it does not lead to problems. If we consider the problem: You have 2 sweets but no learners to divide them among you cannot do anything. We realise that \(2 \div 0\) is not possible. You cannot divide by 0.

ACTIVITY 1

Refer back to our discussions on the laws of addition, subtraction, multiplication and division.

• What are the similarities that you notice between the laws relating to the four operations?
• What are the differences between the laws relating to the four operations?
Commentary

It is essential and useful for teachers to identify the similarities and differences between the four operations. This ability to recognise patterns in mathematics helps us to simplify our processes and solidify our conceptual understanding.

- The commutative and associative laws apply to single operations and they hold for addition and multiplication. They do not apply to subtraction and division.

- The distributive law is a little more complex in that it involves more than one operation. Understanding the distributivity of multiplication over addition and subtraction is the key to solving multiplication and division problems. Multiplication is distributive over addition and subtraction from both the left and the right, as can be seen by the following number sentences:

  - Left: $2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$
  - Right: $(2 + 3) \times 4 = (2 \times 4) + (3 \times 4)$

  - Left: $4 \times (3 - 2) = (4 \times 3) - (4 \times 2)$
  - Right: $(4 - 3) \times 2 = (4 \times 2) - (3 \times 2)$

- Division on the other hand, can only be distributed over addition from the right, in the sense that $(80 + 20) \div 8 = 80 \div 8 + 20 \div 8$.

This is quite a technical discussion, but it is useful for teachers to have a higher level of understanding of the operations and the relationships between them. This will help you to teach learners at the level they are at and answer their questions meaningfully and appropriately. Come back to this discussion later to consolidate this knowledge if necessary. We now discuss the activities of grouping appropriate to the early grades.

There are two ways of conceptualising division. They are known as grouping and sharing. When we give learners word problems using real-life contexts, they learn that these can be solved in either a grouping or a sharing way. This means that, whether we think of 10 divided by 2 in a grouping or a sharing way, the answer will be the same, and only the strategy is different. We must be sure to explain both division strategies to our learners, and so we will aim to discover more about these two ways of dividing by looking at examples of each.

Grouping

Read the following problem and think about how you would solve it.

I have 90 sweets.
I sell them at the cake sale in bags with 6 sweets per bag.
How many bags can I make?

The number sentence for this problem is $90 \div 6$ bags.

But, to work it out, I need to put six sweets in each bag. I will keep doing this until all the sweets are used up and then I count how many bags I can make.
This is known as grouping division. This division strategy is often neglected, though it occurs often in real situations.

In grouping, you have to make groups of objects first. You find the solution by counting the number of groups. This can be done by using repeated subtraction – as an introductory strategy and by using multiples later on.

**ACTIVITY 2**

Look at the illustrations below and explain the difference between them as solutions to the number sentence $10 \div 2 = \Box$.

Two children can get 5 lollipops each

There are 5 bags with 2 lollipops in each bag

**Commentary**

When solving these problems, it is apparent that the number sentence $10 \div 2 = \Box$ is a simple one, that results in the answer 5. However, when we look at the illustrations, we notice that in the first one, there are two learners, whereas in the second one there are 5 packets. This tells us that the number of ‘groups’ is different in each solution. In the first solution, 2 groups of 5 sweets in each group have been made (so 2 learners could get sweets). In the second solution, 5 groups with 2 sweets in each group (bag) (so 5 bags of sweets have been made).
Sharing

In sharing, you can’t make groups of objects because you don’t know how many objects should be in one group because you first have to share out the items to find out how they are shared. When you share, you have to distribute the objects one by one (or using pairs/other groups, when you are ready to share more efficiently and you have a lot of items to share).

Read the following problem and think about how you would solve it.

I share 65 sweets among 7 learners. How many sweets will each learner get?

The number sentence for this problem is $65 \div 7$ sweets. The way in which we do it is we share out the sweets, until they are all given out, and then we find out how many sweets each learner got. This is known as sharing division. Many division word problems are phrased in this way.

When the sweets are shared out in this problem, each learner ends up with 7 sweets and there are 2 sweets left over.

ACTIVITY 3

Watch the video “Division (sharing and grouping)” (5:14 minutes), to see the difference between grouping and sharing.

- What do you notice about the way the teacher demonstrates solving the problem $45 \div 5$?
- What do you notice about the way that the teacher demonstrates solving the problem $27 \div 3$?
- What did you notice about the language the teacher used as she spoke about these problems?
Commentary

It is clear from the language that the teacher uses that there is a distinct difference between the two problems identified in this activity. The context of the problem determines the kind of division strategy that must be used. When you look at the number sentences, they book just look like division problems:

\[ 45 \div 5 = \square \]
\[ 27 \div 3 = \square \]

However, as you listen to the terminology used by the teacher you can tell that the first problem is a sharing problem. The teacher shares out the sweets between 5 learners so that the sweets are divided equally. This means that you know the total number of sweets and you know how many learners (or groups) you have, and you need to work out how many sweets go into each group. When you compare this to the second problem, it is clear that the second problem is a grouping problem. The teacher tells the learners that there are 27 flowers in total, and that the flowers are put into groups of 3. This means that they know the total number of flowers and the number of flowers that go into each group, and they then need to work out how many groups of flowers they will have in the end.

Check your understanding: Multiple Choice

1) Division is:
   A) Commutative
   B) Associative
   C) None of the above

2) \( 7 \div 0 = \)
   A) 7
   B) Not possible
   C) 0

3) In grouping problems, we know:
   A) The total number of objects, and how many groups there are.
   B) The total number of objects, and how many are in each group.
   C) How many groups there are and how many are in each group.

4) In sharing problems, we know:
   A) The total number of objects, and how many groups there are.
   B) The total number of objects, and how many are in each group.
   C) How many groups there are and how many are in each group.

REFLECTION

- Reflect on your own understanding of grouping and sharing.
- Did you understand the difference between grouping and sharing when you were at school?
- Write a personal goal in relation to how you will help learners to grasp the difference between these two types of problems.

Well done you have completed Lesson 5.
In this lesson we will continue looking at division, and the language we use to understand and solve problems. We will investigate division stories and consider how to deal with remainders. Finally, we will move on to learning about sharing leading to fractions. This is an important aspect of division, and one that needs much practice.

### What you will learn in this lesson
- Division stories
- Division with remainders
- Sharing leading to fractions

### Division stories

As learners develop their understanding of division, they will become more confident in recognising the difference between grouping and sharing. By providing learners with a variety of division stories, we will enable them to construct their own understanding of the strategies used to solve division problems.

It is important to encourage learners to read problems carefully in order to work out what the best strategy would be to solve the problem. By identifying the key parts of the division story, learners can then determine if they know how many groups they have or if they know how many objects go in each group. We need to present problems that involve the two different strategies. Think about these problems- learners need to be able to identify information they present in order to identify the strategy needed to solve it:

<table>
<thead>
<tr>
<th>There are 72 biscuits.</th>
<th>There are 85 flowers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I put 4 biscuits in a box.</td>
<td>There are 5 friends.</td>
</tr>
<tr>
<td>How many boxes do I need?</td>
<td>How many flowers should each friend get?</td>
</tr>
</tbody>
</table>

In the first problem, about the biscuits, the following information is given:
- The total number of biscuits
- The number of objects (biscuits) that will go in each group (box)
- The learners need to find out how many groups (boxes) they will need.
- This is a **grouping problem**.

However, in the problem with the flowers, the learners know:
- The total number of flowers
- The number of groups (friends)
- The learners need to find out how many objects (flowers) will go in each group.
- This is a **sharing problem**.
ACTIVITY 1
Make up a story that would lead to a number sentence involving division.

Commentary
As we saw when we discussed multiplication stories, it is important for the learners to be involved in problems that provide opportunities for success. They need to be able to develop their confidence by completing problems that have a similar structure and layout. You need to give problems that require grouping and sharing as solution strategies. Encourage the learners to create their own division stories, thinking about how to use the mathematical language that they have learnt. Make sure that you have modelled the appropriate use of the language so that the learners have a good example to follow. Remember to get learners thinking about whether their problems are sharing problems or grouping problems and ask them to explain their reasoning.

Division with remainders
The underlying concept of division is that we need break down quantities into equal sized groups. Once learners understand this, then remainders do not pose a threat. Learners will realise that remainders are a normal part of life, and they can discuss what they think should happen to the “left over” items (the remainder). Think about the following problem. How would learners solve it?

A farmer has 8 sheep pens, and 79 sheep. During the day, the sheep graze in the veld and at night they are put into the pens. How many sheep will go into each pen?

This is a sharing problem, since the sheep have to be distributed into the pens. This can be done in the following way, allocating sheep to the pens evenly.

But what learners will find, is some sheep are ‘left over’. They will find that they can have 9 sheep in each pen, but then there will be 7 sheep left over. They will need to discuss what they think should happen to the remaining 7 sheep. Should they be left out in the veld on their own? Should one extra sheep be put in each pen so that 7 pens have 10 sheep and 1 pen has 9 sheep? Encourage learners to justify their ideas. In the problem:
Here is another problem learners could solve:

The farmer has 99 eggs. He packs them for market in boxes of 12. How many boxes did he fill?

This is a grouping problem, since the eggs have to packed in groups of a given number. This could be solved using repeated subtraction (or multiples of 10s and 2s).

ACTIVITY 2

Division with remainders in context

Watch the video “Division with remainders in context” (3:05 minutes), to find out how to deal with remainders.

• What did you notice about the learners’ answer to the problem?
• What did you notice about the teacher’s response?
Commentary

In the video, the teacher poses a real-life problem, and asks the learners to help her solve it. She encourages the learners to discuss how they would solve the problem, and to give an answer without using their blocks. The reason the teacher didn’t want the learners to use their blocks was firstly so that they could think logically about the problem. If the learners had used blocks, then they would have just said the answer is 4 with 3 left over. They wouldn’t necessarily have thought about the fact that the ‘3 left over’ were actually learners who would have nowhere to sit. It is important for learners to understand exactly what the question was asking, and to provide an answer to that question, rather than just the numerical facts.

Secondly, the teacher wanted the learners to see how she used the table on the board to help her solve the problem. The teacher shows the learners that they can count in multiples of six to find out how many benches will be needed to hold all 27 learners. The top row shows the number of benches, and the bottom row goes up in multiples of 6. This creates a visual representation that helps learners to see how they should answer the question.

Sharing leading to fractions

Many people find the concept of fractions a bit tricky, and this can result in some maths anxiety for both teachers and learners. However, it is important to note that if you have allowed learners to develop a solid understanding of division using concrete apparatus, this leads well into the learning about fractions.

Up until now, we have discussed division in terms of breaking quantities into equal groups. As we start looking at sharing leading to fractions, we can move towards dividing a shape into equal parts. As we can see from the images below, we can show learners that 1 rectangle can be divided into 2 equal parts.

This is a good time to remind learners that the fractional parts need to be the same size. Posing a problem about dividing a cake fairly provides a context for this discussion. Learners will realise that it would not be fair to cut a cake with slices cut in a variety of sizes. You can use the concept of fairness to stimulate learners to think about equal sized parts. Learners will be quick to point out that all the slices need to be the same size. In reality, it is difficult to cut circular shapes into exactly equal parts but you should try. A loaf of bread could also be used s an example. But paper folding can be done more easily in a class.

ACTIVITY 3

Think about how you can use paper folding to help learners grasp sharing leading to fractions.

- Why is paper folding a good why for learners to learn about sharing and fractions?
- Describe how you could use a paper folding activity to teach learners about breaking a whole into 4 parts.
Commentary

Paper folding is a useful way of demonstrating to learners that one whole (one paper shape) can be divided into equal parts. A benefit of paper folding is that it is very obvious that the parts are all equal sizes. These parts can be folded on top of each to show that they are equal, or they can be cut up and placed on top of each other. This activity emphasises for learners that fractional parts are exactly the same size, and it will give them the opportunity to use the correct mathematical language as they describe their parts. We will discuss more about fractions in the last 3 lessons of this part of the course.

Check your understanding: True or False?

1) Learners need to practice a variety of both grouping and sharing problems.

2) Remainders should not be taught until learners are older.

3) Division problems do not have to be related to real-life situations.

4) Paper folding should only be done with very young learners.

REFLECTION

• Reflect on your own views on remainders.
• Think about whether you felt anxious about dealing with reminders when you were at school.
• Write a personal goal in relation to how you will approach the teaching and learning of remainders in your classroom.

Well done you have completed Lesson 6.
In this lesson, we will investigate how to solve multiplication and division problems that involve higher numbers. This is often quite challenging for learners, so it is important that you as the teacher are comfortable with different methods of solving problems. In this way, you will be able to support learners as they construct their own understanding. In this lesson, we will also look at multiplication and division as inverse operations and spend time considering some common learner errors.

**What you will learn in this lesson**
- Multiplication and division with larger numbers
- Multiplication and division as inverse operations
- Learner errors

**Multiplication and division with larger numbers**

Learners typically find it extremely difficult to multiply and divide large numbers. It is for this reason that we need to ensure that they have developed a sound understanding of number relationships through the use of concrete resources. Learners need a variety of problems so that they can practice a range of solution strategies once they move onto doing numeric calculations.

If learners are able to understand what they are doing when they multiply and divide, then they can find ways to solve problems without relying on learnt algorithms. For example, if learners have an understanding of the laws of multiplication, then they will be able to solve the problem 3 x 28 quite easily. Rather than trying to count in 28s, or trying to write out the multiplication algorithm in column format, they can break up the problem into:

\[
20 \times 3 + 8 \times 3 = \square \\
60 + 24 = 84
\]

This is far easier to solve in your head than the problem 3 x 28.

In the same way, with division, if learners are given the problem 462 ÷ 2 = \square then they can use their knowledge of partitioning to help them solve it more efficiently. Learners can break the number 462 into 400 + 60 + 2, and then divide each part by 2 which can be done mentally.

\[
400 \div 2 = 200 \\
60 \div 2 = 30 \\
2 \div 2 = 1 \\
200 + 30 + 1 = 231.
\]

Give learners as many opportunities as possible to talk about solving problems together and to explain their reasoning to each other while they do so.
ACTIVITY 1

Division of 2-digit numbers

Watch the video "Division of 2-digit numbers" (3:38 minutes), to see how the learners learn to divide 2-digit numbers.

• How did learners use the base ten blocks to help them?
• What other method could learners have used to solve the problem?

Commentary

The learners used their blocks to share out the 39 between 3 groups. In doing this, they could see that $39 \div 3 = 13$ because each group got 1 ten and 3 ones. It was clear that this was a sharing problem, because the learners knew how many groups there were, but they didn’t know how many went into each group. Learners could have used partitioning to solve this problem in a different way. They could have broken down 39 into 30 + 9, and then divided each number by 3.

\[
\begin{align*}
30 \div 3 &= 10 \\
9 \div 3 &= 3 \\
10 + 3 &= 13
\end{align*}
\]

Multiplication and division as inverse operations

Division and multiplication are inverse operations. This means that division “undoes” what multiplication “does”. Where learners have had lots of exposure to times tables they will more easily see that division is the inverse of multiplication. You should encourage learners to build up their division tables alongside their times tables so that they can identify the patterns for themselves.

Give the learners practical examples to solve, letting them think about how they can work out the problems. For example:

<table>
<thead>
<tr>
<th>I have 42 pencils.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I share the pencils between 6 friends.</td>
</tr>
<tr>
<td>How many pencils will each friend get?</td>
</tr>
</tbody>
</table>

\[
\begin{array}{ccc}
& & \text{42} \\
\downarrow & \downarrow & \downarrow \\
\text{6} & \text{?} & \text{?}
\end{array}
\]

If learners think about a part-part-whole model, they will realise that they can solve the problem by simply recalling their 6 times table. It is critical that learners reach the stage where they can use multiplication facts to solve division problems.
**ACTIVITY 2**

Give learners a division problem to solve.

- Watch learners solve the problem and see their level of understanding.
- Ask learners to use multiplication to check their solution and see whether they are able to use the inverse operation to clarify their understanding.

**Commentary**

The learners will need to remember that they need to obey the laws of multiplication and division. This means that the numbers in the number sentences will have to be written in a certain order in order to abide by these rules. Learners can find this tricky, and this is why it is useful to use a part-part-whole diagram to help them remember which number is the ‘whole’ and which number is a ‘part’. For example, if the learners were given the problem:

```
         96
There are 96 flowers.
They are put into 8 vases.
How many flowers must go into each vase?
```

By using a part-part-whole diagram, the learners will most probably recognise that the problem can be solved by using their knowledge of the 8 times table. They can use the diagram to identify the following number facts:

- \( 96 \div 8 = 12 \)
- \( 96 \div 12 = 8 \)

They will realise that asking themselves "\( 42 \div 6 = ?? \)" can be found by remembering that \( 6 \times 7 = 42 \).

**Learner errors**

Teachers need to anticipate possible areas where misconceptions may arise, and division is one topic where some confusion may develop. For example, when teaching learners about sharing leading towards fractions, learners tend to make a common error where they say the number in a group instead of the number of groups. If you anticipate this mistake, then it is easier to address it and re-direct understanding during your teaching, rather than trying to undo learnt ideas. Misconceptions cannot simply be uprooted and replaced with new, “correct” concepts. And they can be difficult to change.

Some learners find it difficult to let go of oversimplified rules that they have learned in the early grades, especially if these seem to be simple and clear. Some misconceptions originate in teachers’ efforts to make content and procedures simple for learners in the early years of schooling. Teachers need to find out what learners have been taught and where necessary to show them what is inadequate or incorrect about what they have previously learned.

**ACTIVITY 3**

A learner in your Grade 3 class made the following mistake:

\[
\frac{1}{3} + \frac{1}{2} = \frac{1+1}{3+2} = \frac{2}{5}
\]

- Explain the mathematical error that the learner has made.
- What explanation would you give to help the learner to rectify the error?
Commentary

The learner has wrongly applied the rule “what you do to the top you do the bottom” and arrived at the incorrect answer. To assist learners to rectify this error the teacher could use concrete materials to demonstrate addition of fractions so that learners see that half a cake (or an apple) plus a third of a cake or apple cannot be two fifths of a cake or apple. The concrete demonstration will make two things clear to the learner. The first is that unlike fraction parts cannot be added (e.g. $\frac{1}{3} + \frac{1}{2}$ cannot be added as they are). The second is that like fractions can be added by adding the number of fraction parts (e.g. $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$).

Check your understanding: Multiple Choice

1) All learners should:
   A) Solve problems in the same way.
   B) Be given opportunities to try different solution strategies.
   C) Be taught one way to solve problems.

2) Partitioning is:
   A) A more time-consuming way to solve problems.
   B) Only used to demonstrate place value.
   C) An efficient method that can be done mentally.

3) Inverse operations:
   A) Can be used to check answers to problems.
   B) Do not need to be taught.
   C) Should only be taught to older learners.

4) Commonly used phrases in the classroom can be:
   A) A way to help learners remember what to do.
   B) Quick and easy signposts while teaching.
   C) A way of creating misconceptions.

REFLECTION

- Reflect on your use of mathematical language in the classroom.
- Think about whether you are creating misconceptions by the way you say things to the learners.
- Write a personal goal in relation to how you plan to monitor what you say in the classroom.

Well done you have completed Lesson 7.

Answers:
Fractions are numbers, and the development of fraction number concept in the early grades lays the foundation for further teaching about rational numbers in later years. Rational number concept involves an understanding of fractions which involves more than just the finding of parts of a whole. Learners need to be exposed to a range of activities and conceptual teaching on fractions as parts of wholes, ratios, decimals and percentages in order to develop fully their understanding of multiplicative reasoning and rational numbers. Much of this learning happens beyond the Foundation Phase but it is important to see the teaching of concepts in the bigger context.

In the next three lessons you will learn about the teaching of fractions. You will be taken through a series of activities that show you how to provide learners with opportunities to work with a wide variety of wholes (continuous and discontinuous) to enable the development of a solid concept of fractions as numbers.

What you will learn in this lesson

- Fractions as parts of wholes – introducing fractions
- The difference between continuous and discontinuous wholes
- Language to use when finding fractions as parts of wholes

Fractions and wholes

Fractions can be used to represent numbers which are not whole numbers. As such, they are slightly more difficult to come to terms with than whole numbers and are taught once basic number concept has been established. The first part of this activity will look in a detailed manner at sound methods for the teaching of fractions to young learners. You should be able to follow these ideas and ensure that all of the information given forms part of your own knowledge. It is vital that all teachers of mathematics have a good concept of fractions themselves.

We need to ensure that learners are given adequate exposure to a great enough variety of examples of fractions in concrete demonstrations so that they are able to form their own abstract concept of what number the fraction numeral represents. We begin by looking at fractions as parts of concrete wholes and progress from there to more abstract working with fractions.

Types of wholes

The first important thing we should stress is that we can find fractions of continuous and discontinuous wholes. These two types of wholes are not always given equal representation in workbooks and activities. We should not emphasise one more than the other or we risk giving an unbalanced idea of concrete wholes.
**Continuous whole**

- Single items that make up the whole also called unit wholes
- Fraction parts are equal sized parts into which the unit whole has been divided
- Examples: an orange, a piece of paper, a slab of chocolate, a circular disc, a loaf of bread

**Discontinuous whole**

- Groups of items that together make up the whole
- Fraction parts are equal sized groups (same number of items in each group).
- Examples: 15 oranges, 6 biscuits, 27 counters, 4 new pencils

**ACTIVITY 1**

- List and describe five of your own examples of continuous wholes.
- List and describe of your own examples of discontinuous wholes.

**Commentary**

When deciding on your own examples, remind yourself of the key features of each type of whole. The continuous wholes should be **one single item** that can be cut/broken up to find the fraction part, and the discontinuous wholes should be **multiple item wholes** (anything more than one) that have to be grouped to find the fraction part. Learners need to be exposed to as many different examples of wholes when working with concrete items or diagrammatic representations in order to be able to develop a well generalised fraction number concept.

**Language to use when finding fractions as parts of wholes**

To assist learners to establish their fraction concept, we must use good language patterns consistently when talking about fractions. It is thought that our language is linked to our thinking, and so by encouraging learners to talk about what they see, we help learners to transfer what they see in the concrete demonstrations into their abstract thought. The language patterns that we are talking about link to the different kinds of wholes that learners encounter when working with concrete (real objects) and semi concrete (drawings of objects) examples.
Language patterns (talking about) continuous wholes

It is essential to use the correct language patterns when teaching learners about fractions. This can be a tricky concept for learners, and they often have misconceptions about the terminology of fractions. For example, when sharing a piece of cake, one learner may complain that “My half is smaller than your half!” This incorrect use of the word half shows that the learner does not understand that fractional parts need to be exactly the same size. Below is an example to show you what it means to use correct language when working with a continuous (single unit) whole.

![Whole disc](image)

To find \( \frac{1}{5} \) of my circular disc, I first divide the whole circular disc into 5 parts of equal size.

![Whole divided into 5 parts](image)

Each part is \( \frac{1}{5} \) of the whole, and if you shade one of these parts, you have shaded \( \frac{1}{5} \) of the whole.

**ACTIVITY 2**

**Unitary fractions of continuous wholes – teacher**

Watch the video “Unitary fractions of continuous wholes – teacher” (4:48 minutes), to see how the teacher demonstrates finding fraction parts.

After watching the video try out the following activities for yourself:

- Find \( \frac{1}{3} \) of the rectangle given below:

![Rectangle](image)

- Shade \( \frac{1}{4} \) of a strip of paper.
- Illustrate and explain how to find \( \frac{1}{6} \) of a circular cake.

**Commentary**

Practice the correct language pattern to use while speaking aloud about finding fraction parts. The use of good language allows learners to develop thinking patterns since it gives learners verbalisation skills that can support abstract thinking. This activity focuses on language patterns when speaking about fraction parts of continuous (single item) wholes.
• To find $\frac{1}{3}$ of the rectangle, I first divide the whole rectangle into 3 parts of equal size. Each part is $\frac{1}{3}$ of the whole, and if I shade one of these parts, I have shaded $\frac{1}{3}$ of the whole rectangle.

<p>| | | |</p>
<table>
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• To find $\frac{1}{4}$ of my paper strip, I first divide the strip into 4 parts of equal size. Each part is $\frac{1}{4}$ of the whole, and if I shade one of these parts, I have shaded $\frac{1}{4}$ of the strip.

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• To find $\frac{1}{6}$ of my circular cake, I first divide the whole circular cake into 6 parts of equal size. Each part is $\frac{1}{6}$ of the whole, and if I shade one of these parts, I have shaded $\frac{1}{6}$ of the circular cake.

Language patterns (talking about): discontinuous wholes

Here is an example to show you what it means to use correct language when working with a discontinuous (multiple unit) whole.

Find $\frac{1}{8}$ of 32 counters  

32 counters (shown above) represent the whole.

I put my counters into 8 groups of equal size. There are four counters in each group.

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</thead>
</table>

One of the groups of equal size is $\frac{1}{8}$ of the whole.
Watch the video "Unitary fractions of discontinuous wholes – teacher" (3:36 minutes), to see how the teacher demonstrates finding $\frac{1}{3}$ of 12.

After watching the video try out the following activities for yourself:

• Find $\frac{1}{3}$ of 27 oranges, as given below.

• Find $\frac{1}{3}$ of 30 marbles.

• Find $\frac{1}{4}$ of 20 cupcakes.

Commentary

This activity focuses on language patterns when speaking about fraction parts of discontinuous (multiple item) wholes. Notice how with a discontinuous whole you work with groups of items according to the fraction you have to find. In the first activity the oranges have been drawn. Learners should draw dots (or other representations) if they need a drawing to help them find the fraction parts.

• To find $\frac{1}{3}$ of 27 oranges, I first divide the oranges into 3 groups of equal size. I find three groups with nine oranges in each group. Each group is $\frac{1}{3}$ of the whole, and so 9 oranges is $\frac{1}{3}$ of 27 oranges.

• To find $\frac{1}{3}$ of 30 marbles, I first divide the marbles into 3 groups of equal size. I find three groups with 10 marbles in each group. Each group is $\frac{1}{3}$ of the whole, and so 10 marbles is $\frac{1}{3}$ of 30 marbles.
To find $\frac{1}{4}$ of 20 cupcakes, I first divide the cupcakes into 4 groups of equal size. I find four groups with five cupcakes in each group. Each group is $\frac{1}{4}$ of the whole, and so 5 cupcakes is $\frac{1}{4}$ of 20 cupcakes.

Check your understanding: True or False?

1) A packet of biscuits is an example of a continuous whole.
2) A circular disc is an example of a continuous whole.
3) 25 beads is an example of a discontinuous whole.
4) It does not matter what the whole is when you look for fraction parts.

REFLECTION

• The concept of a fraction grows from a concrete understanding of a part of a whole to an abstract understanding of a fraction as a number. Reflect on what this means in relation to your teaching of fractions.

• When you introduce fractions to learners do you give them a lot of opportunities to work with concrete manipulatives? Describe some of the activities you do.

Well done you have completed Lesson 8.
Curriculum coverage is never just about completing the learning programme designed by a teacher or school. In this series of lessons on fractions all of the curriculum content of the early grades as well as an extension of some of these concepts is presented. In this lesson we focus on fractions as numbers.

What you will learn in this lesson
- How foundational fraction number concept fits into the development of number concept
- Unit fractions and non-unit fractions
- More about continuous and discontinuous wholes

Teaching fractions as part of the curriculum

Teachers need to think carefully about the levels of understanding of content that are appropriate to the age-level of the learners. This means that it is important to think about the relation between content and concepts. The content of a topic consists of concepts, which organise the information. Therefore, concepts need to be taught (or learned) in a particular order since they are made up of different levels of complexity.

Concepts also regulate what can and should be connected with what, and they give a form or a shape to the content. Concepts give a shape to the information by showing which piece of information is more central than another, which should come before the other, which is bigger and which is smaller etc. In order to know content well, teachers and learners need to understand the concepts that inform or shape it. Fraction number concept develops over time and can be summarised in the following concept map.

What the concept map about fractional thinking shows is that concepts structure the content into a network of ideas.
**ACTIVITY 1**

How should a learner answer this question:

*Which of the following circles has NOT been shaded in halves?*

- A
- B
- C
- D

- What is the correct answer?
- What conceptual knowledge does the learner need in order to get to the correct answer?
- Does this conceptual knowledge ‘fit’ into the concept map above? If so, where?

**Commentary**

To get the correct answer of B, learners need to have an understanding that there are many different ways of showing a half of a whole in a concrete representation. They needed to be alert when reading the question to realise that they had to identify which image did NOT represent a half. Recognising a half of a given circular disc involves recognising that the whole must be divided into two parts of equal size. The half can be made of smaller pieces (such as two quarters) that do make up a half when taken together. This means that it would fit into the concept map above under ‘Introduction – fraction as a part of a whole’.

**Unit fractions and non-unit fractions**

When you introduce fractions to learners, you will begin by finding unit fractions (as we have done above). A unit fraction is a fraction of the form $\frac{1}{n}$. The numerator is one and the denominator can be any number except zero. You must allow the learners to experiment with finding unit fractions of a broad variety of wholes.

At the beginning you will restrict your discontinuous wholes according to the denominator. For example, if the denominator is 6, you will only ask the learners to find fraction parts of 6 counters, or 12, 18, 24, etc. counters (multiples of 6). You must also remember to set tasks involving continuous wholes as well as discontinuous wholes.

Vary your apparatus as widely as you can. Use pieces of paper, string, sand, water, beads, counters, strips of paper, bottle tops – whatever is easily available.

**Continuous whole example of a unitary fraction**

$\frac{1}{4}$ of 1 square = $1 \div 4 = \frac{1}{4}$ of the square. (In the continuous whole example the answer is a fraction, not a whole number.)

**Discontinuous whole example of a unitary fraction**

$\frac{1}{4}$ of 20 stamps is $20 \div 4 = 5$ stamps. (In the discontinuous whole example the answer is a whole number.)

Once you are satisfied that your learners have established the general result: $\frac{1}{n}$ of $m = m \div n$, you can move on to finding non-unitary fractions (these will be discussed later in the session).

You will now set tasks for your learners to find fraction parts of wholes, where the fraction is of the type $\frac{m}{n}$ where $n \neq 0$. This is purely an extension of the previous activities, where you found $\frac{1}{n}$ of a whole. These fractions are called non-unitary fractions. Learners should not experience too many difficulties finding non-unitary fractions if unit fractions have been grasped well. Examples of finding non-unitary fractions of a continuous and then a discontinuous whole are shown below.
### Continuous whole: Find $\frac{5}{6}$ of a square sheet of paper.

<table>
<thead>
<tr>
<th>The whole.</th>
<th>The whole divided up into 6 parts of equal size.</th>
<th>5 of the 6 parts of equal size have been shaded. $\frac{5}{6}$ of the whole has been shaded.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Language pattern**

The whole is a square sheet of paper. I fold the whole up into six parts of equal size in order to find sixths. Each part is $\frac{1}{6}$ of the whole, so 5 of the six equal sized parts is $\frac{5}{6}$ of the whole.

### ACTIVITY 2

**Non-unitary fractions of continuous wholes – teacher**

Watch the video “Non-unitary fractions of continuous wholes – teacher” (4:52 minutes), to learn more about non-unitary fractions of continuous wholes.

- Illustrate $\frac{2}{3}$ of a pizza.
- Shade $\frac{3}{5}$ of a rectangular sheet of paper.

**Commentary**

When you do these activities, make an effort to use the full, correct language when you speak about finding fraction parts of the various given wholes. Notice how with a continuous whole you cut/break/divide the whole according to the fraction you have to find. Since you must find a non-unit fraction, you must take more than one of the equal parts into which you divided the whole.

- To find $\frac{2}{3}$ of my pizza, I first divide the whole pizza into 3 parts of equal size. Each part is $\frac{1}{3}$ of the pizza, and if I shade two of these parts, I have shaded $\frac{2}{3}$ of the pizza.
- To find $\frac{3}{5}$ of my rectangular sheet, I first divide the whole rectangular sheet into 5 parts of equal size. Each part is $\frac{1}{5}$ of the rectangular sheet, and if I shade three of these parts, I have shaded $\frac{3}{5}$ of the rectangular sheet.
**Discontinuous whole: Find \( \frac{3}{4} \) of 36 marbles**

The whole.

The whole divided into quarters.

Three of the four groups (representing \( \frac{3}{4} \) of 36) have been shaded.

<table>
<thead>
<tr>
<th>Language pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>The whole is 36 marbles. I divide the whole up into four groups of equal size in order to find quarters. There are 9 marbles in each group. One group of 9 is ( \frac{1}{4} ) of 36, and so 3 groups of 9 are ( \frac{3}{4} ) of 36, so 27 is ( \frac{3}{4} ) of 36.</td>
</tr>
</tbody>
</table>

**ACTIVITY 3**

Watch the video "Non-unitary fractions of discontinuous wholes – teacher" (5:40 minutes), to find out more about non-unitary fractions of discontinuous wholes.

After watching the video try out the following activities and write out the full language pattern you would use in each case, so that you can check your own ability to talk fluently about the fraction parts you are finding and model it for learners.

- Show how you find \( \frac{2}{5} \) of 20 bricks.
- What is \( \frac{3}{4} \) of 20 liquorice strips?
Commentary

When you do these activities, make an effort to use the full, correct language when you speak about finding fraction parts of the various given wholes. Notice how with a discontinuous whole you work with groups of items according to the fraction you have to find. Since you must find a non-unit fraction, you must take more than one of the equal groups into which you divided the whole.

- To find $\frac{2}{5}$ of 20 bricks, I first divide the bricks into 5 groups of equal size. I find five groups with 4 bricks in each group. Each group of 4 is $\frac{1}{5}$ of the whole, and so 8 bricks is $\frac{2}{5}$ of 20 bricks.

- To find $\frac{3}{4}$ of 20 liquorice strips, I first divide the liquorice strips into 4 groups of equal size. I find four groups with five liquorice strips in each group. Each group of 5 is $\frac{1}{4}$ of the whole, and so 15 liquorice strips is $\frac{3}{4}$ of 20 liquorice strips.

You could turn some of your fraction finding into games or activities. In this way, you could keep the learners busy for slightly longer periods of time, while they are learning and discovering ideas in an interesting and enjoyable way. In this example, learners are given 20 counters. They must then try to find all the possible fraction parts that they can, of 20 counters. They could work in groups of two to four members (not more, as they would not have enough of a chance to express themselves). The discussion of the different fraction parts could go on in the whole group. Once the group thinks that they have found all the possible fraction parts they can put up their hands and say “Full House!”, to call you to come and check up on them. As a follow up, ask each learner to record in full and good language one of the fraction parts which they found. Try this activity out yourself!

Check your understanding: True or False?

1) $\frac{3}{4}$ is a non-unitary fraction.

2) A piece of paper is an example of a discontinuous whole of which you could find a half.

3) $\frac{3}{4}$ of 24 apples is 6 apples.

4) You can use examples of incorrectly shared fractions when you teach learners about fractions.

REFLECTION

• What is the benefit of giving learners many varied examples of whole when teaching them about fractions?

• What is the benefit of giving learners many varied examples of fractions to find when teaching them about fractions?

• Write a personal goal about the use of manipulatives when teaching fractions in your class.

Well done you have completed Lesson 9.

Answers:

1) True

2) False. A piece of paper is an example of a continuous whole, but you can find a half of the piece of paper by folding it into two equal sized parts.

3) False. $\frac{3}{4}$ of 24 apples is $3 \times 6 = 18$ apples.

4) True. You can use such examples to give learners an opportunity to reject what you show them as a correct shading. This helps you check their understanding.
In the early grades, learners are not expected to learn a lot of fraction terminology, but as a teacher of maths you need to know more than you are expected to teach so that you could answer questions asked by learners and go beyond the basic curriculum expectations. In this lesson some important fraction terminology is introduced to develop your knowledge of fractions. There is also input on equivalent fractions and comparing fractions. Finally, you will be given the opportunity to think about dealing with learner misconceptions about fractions.

What you will learn in this lesson

- Fraction numerals, like and unlike fractions, proper and improper fractions
- Equivalent fractions
- Comparing fractions
- Working with fractions misconceptions

Fraction numerals

Show learners how to write a fraction numeral and tell them the terminology. Make sure they know which of the numerals is the numerator (the number at the top of the fraction numeral) and which is the denominator (the number at the bottom of the fraction numeral).

You must learn these names if you do not already know them. This is important terminology in the section of fractions. Make sure that learners use the terminology repeatedly, to help them build the words into their regular speech.

Like and unlike fractions:

We call fractions which have the same denominators like fractions. Fractions whose denominators are not the same are called unlike fractions. For example \( \frac{3}{7}, \frac{5}{7} \), have 7 as their denominator.

Proper and improper fractions

When the numerator of a fraction is smaller than the denominator of a fraction, the fraction is called a proper fraction. When the numerator of a fraction is bigger than the denominator of a fraction, the fraction is called an improper fraction.
Equivalent fractions

Your learners will already have begun to notice certain equivalent fractions before you consciously introduce the topic in class. They might have begun to say things to you like “but \(\frac{2}{4}\) is the same as \(\frac{1}{2}\)”. You should encourage this early observation even if it is not called for in curriculum specifications. You could possibly even comment that they have noticed an important quality that they will learn more about later. Here is an activity that you could use to help clarify the understanding of equivalent fractions using concrete wholes.

**ACTIVITY 1**

Take 5 pieces of paper that are the same size. Fold each of them into thirds using vertical folds, as illustrated below. Shade in the first third on each piece of paper.

A  B  C  D  E

Now fold pieces B, C, D and E using horizontal folds as indicated below:

A  B  C  D  E

What fraction of each piece of paper has been shaded?

**Commentary**

The fraction represented by the shaded part on each piece of paper is the following:

- A \(\frac{1}{3}\) is shaded
- B \(\frac{2}{6}\) is shaded
- C \(\frac{3}{9}\) is shaded
- D \(\frac{4}{12}\) is shaded
- E \(\frac{5}{15}\) is shaded

All of these fractions have the same value (one third or \(\frac{1}{3}\)) although they are written in different ways with different fraction numerals and spoken about using different fraction names (for example two sixths or \(\frac{2}{6}\), and so on).

**Comparing fractions**

It is natural to compare whether or not certain numbers represent more or less than other numbers. When we do so for fractions, this process is sometimes fairly involved. In early grades, comparisons of fractions are based on concrete representations, which lays the foundation for work in later grades with purely numeric examples. When we compare fractions, we often ask or need to find out “which one is greater and by how much?” The solution to this is not always as clearly evident as it is in whole number questions. Here is an example of such a comparison.
ACTIVITY 2

Let’s compare $\frac{2}{3}$ and $\frac{3}{4}$, using a concrete discontinuous whole. Let 12 flowers be our whole.

The whole

$\frac{2}{3}$ of 12 is 8

$\frac{3}{4}$ of 12 is 9

Which fraction is greater and by how much?

Commentary

You should do this activity with concrete material yourself – use counters to represent flowers, as drawings are static and much less convincing (or satisfying) when doing introductory examples. To work out which fraction is greater we need to look at the number of flowers we found for each fraction part. If you do this concretely, you should find that $\frac{3}{4}$ is greater than $\frac{2}{3}$ by one flower.

But what is the value of one flower? If there are 12 flowers in the whole, then one flower is $\frac{1}{12}$ of the whole. We can thus say that $\frac{3}{4}$ is greater than $\frac{2}{3}$ by $\frac{1}{12}$.

Fraction walls can also be used to compare the sizes of different fraction parts. You could show your learners this fraction wall and ask them the following questions. They should answer with reference to the fraction wall to support their reasoning about the relative sizes of given pairs of fractions.

<table>
<thead>
<tr>
<th></th>
<th>$\frac{1}{2}$</th>
<th>$\frac{1}{2}$</th>
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<tbody>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
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<td>$\frac{1}{5}$</td>
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<tr>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

Misconceptions and fractions

Misconceptions are part of the knowledge that learners develop, and so form part of their current knowledge. Teachers need to build on learners’ current knowledge. This means that they need to listen to, work with and build on learners’ misconceptions as well as on their correct conceptions. Learning always involves transforming current knowledge. Therefore, if misconceptions exist, the existing knowledge will need to be transformed and re-structured into new knowledge. If teachers listen to and work with learners’ thinking they can learn about:
• the types of conceptions learners have;
• how to build onto these conceptions;
• how to use misconceptions to transform learners’ thinking and to inform teaching.

Here is an activity that gives you an opportunity to think about learners’ misconceptions in relation to fraction concept and how to work with these misconceptions in a constructive way. This will help learners come to correct conceptualisation of the concepts.

ACTIVITY 3

This multiple-choice question about fractions (and the way learners answered it) gives insight into learners’ thinking about fraction number concept.

Tshepo has a slab of chocolate with 32 pieces. He broke off a quarter of the slab. Which of these drawings shows a quarter of the slab of chocolate?

- First think about the correct answer to the question yourself.
- In a sample of learners that answered the question in a test they had, 25% chose the correct answer (A) while 31% chose the incorrect answer (B). There were also learners that chose the other two incorrect options.
- What does this show you about the way learners understand pictorial representations of fractions?

Commentary

The correct answer to this question is A, because in this case the number of blocks in the whole (32) has been divided into four equal parts of 8. 8 blocks is a quarter of 32 blocks.

Answer B – the most highly chosen (incorrect) answer, was probably chosen because it “looks” most like the fraction parts into which learners have most often seen shapes being divided. The other shapes, given as potential quarters have unusual, irregular shapes. B was probably chosen because the learners saw it as a more familiar shape (a rectangle). This means they did not count the individual blocks since there are only 6 blocks in this rectangle which is not one quarter of 32. Learners are often not exposed to fraction parts which are irregular in shape. The familiarity which might have influenced their choice here disadvantaged the learners.

The misconception seen here is called an over-generalisation. Learners probably chose it because it is the most familiar type of “quarter” of the four options, based on the generalisation that when a whole is divided into quarters they will look like this. They did not evaluate the size of the given part (six pieces) in relation to the whole (which was said to have 32 pieces). This results when learners have not been shown enough of a variety of different ways in which wholes can be broken up into parts.
Check your understanding: Multiple Choice

1) An improper fraction is:
   A) When the numerator of a fraction is smaller than the denominator of a fraction.
   B) When the numerator of a fraction is bigger than the denominator of a fraction.
   C) When the numerator of a fraction is the same size as the denominator of a fraction.

2) Equivalent fractions should be:
   A) Discussed naturally as learners ask questions.
   B) Avoided at all costs.
   C) Only covered at the appropriate time in the curriculum.

3) Learning should:
   A) Happen in such a way that misconceptions are impossible.
   B) Ignore minor misconceptions.
   C) Always involve transforming current knowledge.

4) Over-generalisations occur when:
   A) Learners have not been shown enough of a variety of methods.
   B) Learners have been shown too many different methods.
   C) Learners are guessing at the answers.

REFLECTION

• Reflect on why it is important for you as a teacher to have knowledge that goes beyond what you have to teach your early grade learners.

• Write a personal goal in relation to ways in which you could build your own knowledge of fractions so that you can give learners the best possible learning experience in your classroom.

Well done you have completed Lesson 10.
In Part 4 you will be introduced to the topics Space and shape, Measurement and Data handling. The focus on play-based learning, interactive teaching and using error analysis to inform your teaching will continue. In this module you will first learn about the teaching of Space and shape - one of the core topics of early grade maths learning that enables learners to orientate themselves in space, learn the language related to the topic and start to reason about spatial concepts. You will then learn about the teaching of the measurement topics - time, length, mass, capacity and volume. Finally you will learn about the teaching of Data handling, where a focus on the data handling cycle and how this teaching can be integrated with many of the other concepts taught in the early grades will be introduced.
In this lesson, we will discuss the topic Space and shape, outlining the intended outcomes for early grade learners. We will also learn about Van Hiele’s Levels of Geometric Thought which is a theory on how learners learn about Space and shape. We will then look at 2-D shapes, considering what makes up shapes, and what makes one shape the same as or different from another. We will use this investigation to help us with the process of naming and recognising 2-D shapes.

There are many people who are unfamiliar or uncomfortable with this field of mathematics. They sometimes see it as inaccessible and impossible to master. If you are one of these people (to whatever degree) then hopefully the Space and shape lessons covered in Part 4 will help you to consolidate your understanding of the topic, and to develop an enthusiasm for teaching it.

What you will learn in this lesson
- The goals of the Space and shape topic
- Van Hiele’s Levels of Geometric Thought
- Naming and recognition of 2-D shapes

The goals of the Space and shape topic

Geometry (or Space and shape as we now call it) focuses on the teaching and learning of lines, points, shapes and surfaces. As part of this topic, it is essential to understand the terminology related to the naming, recognition, sorting and classification of shapes (both 2-D and 3-D). There are four main outcomes for the topic of Space and shape, and these are covered across various levels of education.
ACTIVITY 1

Look at the pictures below.

• Can you describe each picture?
• What characteristics do you notice?

Commentary

Whether you could name the shape or not and whatever characteristics of the shape you could give, if you look again you will quickly notice (if you have not already) that some of the shapes are flat and some of the shapes protrude into space. All these drawings are on a flat page – showing learners the actual objects will help them to understand the difference between shapes that are flat and those that are not.

This is the first major distinction that we are going to make in terms of geometric figures – some are called plane figures (they are flat and lie in a plane or flat surface; examples are B, C, E, F, H, I and J above). Others are space figures (they are not flat and protrude from the surface on which they are resting; examples are A, D, G and K above). Plane and space are separate from the dimension of the shape – they tell us whether the shape is flat or not.

Van Hiele’s Levels of Geometric Thought

This theory provides a major contribution to our understanding of how learners learn about geometric concepts. The model is divided into five levels, which are unrelated to our school grades. The levels are not age dependent, but they are sequential and describe a hierarchy of thinking processes. This is an important aspect to remember, as it means that a learner cannot progress to a new level of understanding without having moved through the previous levels first. It also means that you could have different learners in your class operating on different levels of understanding.

Within each level, the focus is separated into two parts called the objects of thought and the products of thought. The products of thought in each level is the same as the objects of thought in the next level, which indicates the progression of learning. Learners need many opportunities to be involved with activities at their current level of understanding. By doing this, learners consolidate their existing understanding whilst starting to explore the ideas of the next level.
As with other mathematical concepts, language is extremely important in the development of learners geometric understanding. If the language used is at a higher level than the learner can cope with, then they will be unable to understand the concept being developed. It is possible to memorise facts about shapes without fully understanding the concepts or content. Learners need to be encouraged to talk about their learning so that they can clarify their knowledge as they share their ideas. The levels do not fit grade levels neatly, but could be categorised in the following way, to show the progression from one level to the next.

**Van Hiele’s Levels of Geometric Thought**

<table>
<thead>
<tr>
<th>Level 0 – Visualisation (Gr R – 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognising and naming shapes</td>
</tr>
<tr>
<td>Working and playing with 2-D and 3-D shapes</td>
</tr>
<tr>
<td>See the shape as a whole: sorting, tracing, matching and drawing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 1 – Analysis (Grades 1 – 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties of shape</td>
</tr>
<tr>
<td>Talking about shapes</td>
</tr>
<tr>
<td>Verbalisation (social and academic language)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2 – Informal deduction (Grades 4 – 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classifying</td>
</tr>
<tr>
<td>Identifying common properties</td>
</tr>
</tbody>
</table>

| Level 3 – Deduction (High School and further education) |

| Level 4 – Rigor (High School and further education) |
ACTIVITY 2
Look at the shapes below.

- How are the shapes similar?
- How are the shapes different?

Commentary

In Level 0 of Van Hiele’s levels of Geometric Thought, the objects of thought are shapes and what they look like. Learners would look at the shapes in the picture and be able to recognise and name them. Learners would focus on sorting and classifying the shapes based on their appearance. The products of thought are groups of shapes that seem alike. Learners need multiple opportunities to observe and feel shapes, taking them apart and building them back up again. Learners can discuss the properties informally, using their own language, without using all of the correct terminology.

It is important to teach at the learners level, however almost any activity can be adapted so that it covers two levels. You should ensure that learners have opportunities to use physical materials and to draw shapes at every level. Let learners sort the shapes according to their own criteria, looking for numbers of faces, edges and vertices, straight sides, round sides, and combinations of these.

Naming and recognition of 2-D shapes

In the early grades, language acquisition takes place at the same time as conceptual development. You always need to be aware of how language is being used in the classroom, and you should listen to and observe learners closely to work out what you think is best for them. There are many different ways to approach the teaching and learning of shapes, and variation is often the key.

Shapes that are 2-D (two-dimensional) are found in a plane. This means that they are flat and that they do not take up space. The CAPS makes a clear distinction between 2-D and 3-D shapes by referring to those that are 2-D as ‘shapes’ and 3-D as ‘objects’. In fact, all objects in mathematics, can also be called shapes. The two dimensions refer to length and breadth or width. Early grade learners should explore the properties of the following 2-D shapes: circles, squares, triangles and rectangles. Learners should be encouraged to participate in activities that involve recognising, naming and describing 2-D shapes. They should also become accustomed to comparing and sorting shapes and using them to create other shapes.

Shapes are interrelated, and when learners become aware of this, they will start to see the shapes as less static and rigid, which is an important progression in their awareness of shapes. You could use paper-folding activities in your teaching on 2-D shapes. This would be a hands-on type of activity through which the learners could discover the characteristics of and interrelations between shapes. Try this out and then record what skills and opportunities you think paper-folding exercises would offer to your learners. Early grade learners should be allowed to take time to establish that a square is always a rectangle and a rectangle is sometimes a square.
**ACTIVITY 3**

Watch the video "Naming 2-D shapes" (3:26 minutes) where the teacher discusses a triangle with her Grade 2 learners.

- How are the learners involved during this activity?
- What language is being developed in the activity?
- What do you notice about the descriptions of the triangle?

**Commentary**

As we know, language is extremely important for learners understanding of mathematical concepts. Learners need to know the necessary terminology and be able to use this terminology to describe and define shapes.

You could investigate relationships between the following shapes in order to establish the truth of the following statements (note that this kind of knowledge would be useful for you as a teacher of mathematics and some learners may be also able to tap into this level of analysis but you would not teach it directly in the early grades):

- A square is always a rectangle.
- A square (and a rectangle) is always a trapezium.
- A rectangle is always a parallelogram.
- A parallelogram is sometimes a rhombus, when it has all four sides equal in length.
- A parallelogram is sometimes a kite, when it is a rhombus.
- A kite is sometimes a square, when both pairs of adjacent sides are equal in length and opposite angles are right angles.
- A rhombus is sometimes a square, when it has right angles.

In the topic Space and shape, a polygon is a simple, closed plane shape made only of line segments. Polygons can have many sides (line segment edges), and so they are named according to the number of sides that they have. Polygon means “many-angled”. The corners of a polygon are called vertices. The number of sides of a polygon corresponds with the number of angles of that polygon.

Mathematical definitions involve the use of mathematical terminology. Therefore, learners need to know the correct terminology and use it often. If the learners in your class are having difficulty with mathematical definitions – check and see if they understand each of the terms involved in the definition and you may find the root of their problem. Be very sure that you always use the correct mathematical language yourself, so that through listening to you, your learners will become acquainted with the terminology: you have to set the example! As mentioned in parts 1, 2 and 3 of this course, knowing the mathematical language and fluency of language (and procedures) is critical aspect of knowing and doing mathematics.
### Check your understanding: Multiple choice

1) Which one of these is not one of the goals of the Space and shape topic:
   - A) Look at shapes in the environment.
   - B) Fold paper to understand symmetry.
   - C) Look at the properties of 2-D and 3-D shapes.

2) In Van Hiele’s Levels of Geometric Thought, there are:
   - A) Six levels
   - B) Four levels
   - C) Five levels

3) The level that most Gr R – 3 learners will operate on is:
   - A) Level 0
   - B) Level 1
   - C) Level 2

4) The dimensions used in 2-D shapes are:
   - A) Height and depth
   - B) Width and depth
   - C) Width and length

### REFLECTION

- Reflect on your experience of learning about 2-D shapes at school.
- Think about how you teach 2-D shapes to your class.
- Write a personal goal with regard to helping the learners in your class to recognise and name 2-D shapes.

### Well done you have completed Lesson 1.
In this lesson, we will discuss using 2-D shapes in geometric patterns. We will revisit our knowledge of patterns, which was covered in Parts 1, 2 and 3, and we will extend this understanding to include patterns involving shapes. We will also broaden our learning of patterns to include tessellation, and we will discover more about the use of tessellations in everyday life. Finally, we will investigate sorting and comparing 2-D shapes by looking at the features of shapes.

What you will learn in this lesson

• Geometric patterns
• Tessellation
• Sorting and comparing 2-D shapes

Geometric Patterns

As we have discussed in Parts 1, 2 and 3 already, patterns are an important part of Maths. Learners need to develop their ability to recognise and understand patterns so that they can use these to help them solve mathematical problems simply and efficiently. A pattern is a sequence of repeating objects, shapes or numbers. Patterns have rules which help us to determine which object, shape or number belongs in the pattern and which do not belong to the pattern.

In the pattern above, we can see that three horizontal lines follow three vertical lines each time. This makes it easy to determine that the next figure in the pattern will be three vertical lines. Patterns are all around us in our everyday lives, and it is helpful for learners to become confident in identifying these patterns. They can look for patterns in the paving, in the trees, on the floor, in the windows or in their clothes.

There are different types of patterns, and it is necessary for learners to recognise these patterns so that they can identify and apply the rules in order to complete the patterns. When completing patterns, learners will need to do some guesswork at first, but then they can use the rules to check if the pattern has been completely correctly.
The first type of pattern is the repeating pattern, where the rule keeps repeating over and over. The most important aspect to identify in a repeating pattern is the 'unit' (also called the core) and the number of elements in the unit because this is the part that repeats.

![Repeating Pattern Example]

The second type of pattern is a growing pattern, where the figures increase as the pattern progresses.

![Growing Pattern Example]

The third type of pattern is a shrinking pattern. In this pattern, the figures decrease as the pattern progresses.

![Shrinking Pattern Example]

It is important to note how growing (and shrinking) patterns start and how they grow (or shrink) by a constant change or by a changing amount.

**ACTIVITY 1**

Watch the video “Geometric patterns” (3:14 minutes) where the teacher discusses patterns with her Grade 2 learners.

- What type of pattern is being used in the lesson?
- What could you do to vary the types of patterns being used in the lesson?
- Why do you think these types of patterns are important in Maths?

**Commentary**

In the video, the patterns demonstrated are repeating patterns. The learners constructed their own patterns, using colour and number of blocks to establish the rules for the patterns. It was a good idea to get the learners to construct their own patterns, as this gave them the opportunity to think about what they understood about patterns, and to create their own rules. By explaining their pattern to the teacher, they had to check that their rule actually worked for the whole pattern.
Perhaps the teacher could have extended the activity by encouraging learners to use a growing or a shrinking pattern, and to think about how these would change their rules. Working with patterns will help learners to develop their understanding of numerical patterns. Numerical patterns can be a sequence of numbers that involve calculations. For example in the pattern:

\[ 12 \quad 17 \quad 22 \quad 27 \quad 32 \]

The rule is that you must add 5 to each number.

The identification of rules and patterns helps learners to simplify the solving of problems, because they become able to find solutions (or parts of solutions) mentally. They do not need to labour through solving each step, because they have an existing knowledge of number facts and patterns on which they can draw.

**Tessellation**

Tessellation is the art of covering an infinite surface without leaving any gaps between the shapes used to cover the surface. An example of this is the way we tile a floor or wall. If a single shape can be used to cover an infinite surface, then we say that that shape can tessellate. For example, squares can tessellate. If we cover a surface using a pattern that involves more than one shape, we call that a multiple shape tessellation. Tessellations can be made out of simple geometric shapes but also out of complex and creative shapes. Historically these patterns and designs go back as far as 4000BC. There is lots of evidence of them in Moslem and Islamic cultures in tapestries, tiles, rugs and quilts.

Give learners multiple opportunities to work with shapes and to make patterns. The properties of shapes become more apparent as learners fit them together, matching and rotating them to create patterns.

**ACTIVITY 2**

Think about tessellation and shapes that can tessellate.

- Will any triangle tessellate?
- Will any quadrilateral tessellate?
- Will any polygon tessellate?
- Experiment with tessellations of polygonal shapes to find out which of them tessellate. Cut out about six of any polygon that you wish to tessellate and see if it will tessellate. Paste down your tessellations on paper and keep them for future reference.

**Commentary**

As we have learnt, tessellation involves placing shapes together without any gaps to cover an infinite surface. It is interesting to note that only a limited number of shapes can form regular tessellations, without assistance from other geometric gap-fillers. Here are some examples of simple shape tessellations and some interesting tessellations drawn by young learners.
There are shapes that cannot tessellate by themselves. For example, circles and ovals do not have angles, and you can clearly see that it is impossible to put a sequence of these next to each other without leaving gaps. With shapes like these, you would need to add in other shapes in order to create a tessellation. Look around for some examples and share them with your learners to encourage them to experiment using shapes.

**Sorting and comparing 2-D shapes**

Learners need to be given many different opportunities to establish their understanding of the similarities and differences between 2-D shapes. It is essential that they verbalise their observations, and so use of the correct terminology should be encouraged. You can ask questions such as “What can you tell me about these shapes?” An open question such as this allows for the learners to develop their thinking and reasoning skills, and to use verbalisation to clarify their own thinking. This process will broaden learners understanding of the Space and shape concepts. They may provide responses such as:

- “That one is smaller.”
- “Both have 4 corners.”
- “The bottom and the top match.”
- “They both have 4 sides.”

Look at the list of terms below and see whether you know all of them. As a teacher you always need to know more than your learners do.

- **Line segment** - A line segment is straight and has two fixed endpoints.
- **Angle** - An angle is formed where two lines meet at a shared point. Angles are measured in degrees.
- **Vertex / Vertices** - A vertex is a point where two lines meet.
- **Regular** - For mathematical shapes it does not mean “common” or “often seen”. If a shape is regular, it means that the sides of the shape are all equal in length and the angles of the shape are all equal in size.
- **Polygon** - A shape with at least 3 straight sides and angles.
- **Quadrilateral** - A shape with four straight sides.

**ACTIVITY 3**

Think about what you know about 2-D shapes.

- What is the value of playing with, building with and drawing 2-D shapes?
- Describe one activity where learners can get this experience in the early grades.

**Commentary**

The terminology of Space and shape is one area in which you need to be confident. Giving learners opportunities to play with shapes while they talk about them will give them the opportunity to become familiar with the shapes and learn the terminology. One such activity could be a game of “Shape Snap” - where images and names of shapes are on the cards and learners play in pairs.

You can also encourage learners to interact with shapes and to develop their verbalisation skills by playing games such as Shape Bingo or Guess My Shape. For Shape Bingo, learners would have a bingo board with shapes on it. They need to listen to the explanations of the shapes and match it to the shapes on their board. The first learners to cover a whole row of shapes shouts
out “Bingo!” To win the game. In Guess My Shape, learners would have to describe a shape hidden inside a bag or packet using only their sense of touch. The other learners would need to guess which shape it was, and the first learner to guess correctly would be called up to describe the next shape. Learners can also use a pictograph to sort and compare shapes [see Lesson 10]. They can arrange the shapes into the circles according to colour, size, how many sides, how many vertices, or whether they have straight or curved sides.

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**Check your understanding: True or false?**

1. Patterns have rules which help us to determine which object, shape or number belongs in the pattern and which do not belong to the pattern.  

2. A jumping pattern is one of the different types of patterns that learners learn about.

3. A circle is a shape that can create a regular tessellation.

4. An angle is formed where two lines meet at a shared point.

---

**REFLECTION**

- How confident are you with regards to your own knowledge of 2-D shapes and their properties?
- What could you do to extend your own understanding and to promote effective learning opportunities in your class?

---

**Well done you have completed Lesson 2.**

**Answers:***

1. True  
2. False – A growing pattern is one of the different types of patterns that learners learn about.  
3. False – A circle needs other shapes with it in order to tessellate.  
4. True
In this lesson, we will move on to looking at 3-D shapes. We will once again look at the naming and recognition of these shapes, considering whether the 3-D objects slide or roll as we investigate the features of 3-D shapes. As part of our discussion, we will continue to emphasise the importance of language and we will look at ways to get learners to actively participate in constructing their own understanding. We will also look at transformations, where we explore how shapes turn, flip or slide.

What you will learn in this lesson

- Naming and recognition of 3-D shapes
- Balls and boxes
- Transformations

Naming and recognition of 3-D shapes

As we move on to 3-D shapes, it is important to remember that 3-D shapes (or three-dimensional objects) take up space. They are not flat. The CAPS refers to these types of shapes as ‘objects’, however they can also be called shapes. The 3 dimensions refer to length, width and height or depth. It is necessary to give learners time to work with 3-D shapes in a variety of ways so that they can experience these dimensions for themselves.

Early grade learners should explore the properties of the following 3-D objects: prisms, spheres, cylinders and cones. You need to provide multiple activities that involve recognising, naming and describing 3-D objects. It takes time and practice for learners to be confident in these skills, and so they need to use physical resources to construct their understanding. Learners also need to practise comparing and sorting 3-D objects and building with these 3-D objects.

A 3-D shape is not a plane figure (a 2-D shape) but rather a ‘space figure’, because it takes up space rather than lying flat in a plane. These shapes have height which makes them protrude up above the plane in which they lie. They can be solid, made of surfaces, hollow skeletons (also called frameworks), or just simply a collection of points which are not flat.

![Solid](image1)  
**SOLID**  
(shaded)  
3-D  
e.g. wood

![Hollow](image2)  
**HOLLOW**  
(lines and dashes)  
3-D  
e.g. cardboard

![Framework](image3)  
**FRAMEWORK**  
(lines)  
3-D  
e.g. wire

![Discrete Points](image4)  
**DISCRETE POINTS**  
(dots)  
3-D  
e.g. dust cloud
The closed space figures that are made entirely of flat surfaces (such as cardboard or paper) are called polyhedra, or sometimes they are called polyhedrons. Polyhedra are three-dimensional, and are made entirely of:

- **Faces** – the flat surfaces (which are all polygons);
- **Edges** – where the faces meet (which are all line-segments);
- **Vertices** – where the edges meet (which are all points).

3-D shapes can be cut open and laid out into a flat 2-D shape. These flat 2-D shapes are called nets, and they can be folded up to make the 3-D shape. We can make nets for all of the polyhedra. There are also nets for some other space shapes which are not polyhedral (such as cones). It is a good idea to allow learners to work with nets, as this helps them to see how the 3-D shapes are constructed, and it also helps them to see how 2-D and 3-D shapes are connected.

### ACTIVITY 1

**3-D Objects**

Watch the video “3-D Objects” (2:48 minutes) where the teacher helps the Grade 2 learners to recognise and name 3-D shapes.

- How are the learners involved during this activity?
- What language is being developed in the activity?
- What do you notice about the descriptions of the 3-D shapes?

**Commentary**

In this video, the teacher is teaching the learners the correct terminology for the 3-D shapes. She is describing the shapes for them as she encourages them to find the shapes in the classroom. It would be helpful to allow the learners the chance to describe the shapes for themselves, as this would help them to better associate the new terminology with the shapes.

Read the descriptions below and think about how confident you are in describing these 3-D shapes correctly.

- A **cube** is a 3-D shape with 6 equal faces.
- A **cone** is a 3-D shape with a circular base.
- A **pyramid** is a 3-D shape with a flat base that could be triangular, rectangular, square.
- A **rectangular prism** is a 3-D rectangular-based prism.
- A **cylinder** has two flat round ends and a round tube. It can roll.
- A **triangular pyramid** is a 3-D triangular based pyramid also known as a tetrahedron.
As we discussed with 2-D shapes, you can use games such as Shape Bingo or Guess My Shape to develop learners' verbalisation skills. The learners would play the games in the same way as described in Lesson 2 except they would now use 3-D shapes rather than 2-D shapes. Learners can also use a pictograph to sort and compare 3-D shapes in the same way as they did for 2-D shapes. They could compare and sort 3-D shapes based on size, colour, curved or straight surfaces, or whether they roll or slide. Learners enjoy opportunities to build using 3-D objects. This type of construction, where they build using concrete materials, allows learners to explore the properties of 3-D shapes in a different way.

**Balls and boxes**

Using real-life 3-D objects gives learners a chance to explore the properties of 3-D shapes through physical experiences. Give each group some boxes and some ball shapes and encourage them to build towers with the items. As learners participate in the activity, they will discover that some objects cannot be placed underneath other objects. For example, they cannot put a ball underneath a box, because the box will fall down. It is important that learners discuss their constructions, and that they verbalise their predications in terms of the stability of each construction.

Learners can experiment to see whether it is possible to make towers by using only balls and only boxes or if a mixture of the two kinds of shapes would also work. They can also vary the height of their constructions, investigating whether or not the height of a tower influences its stability.

**ACTIVITY 2**

Watch the video “Slide and roll” (3:23 minutes) where a Grade 1 class investigates a variety of 3-D objects to determine if they slide or roll.

- Describe an activity where learners can investigate whether 3-D objects slide or roll.
Commentary

A practical activity would be best for this type of investigation, so you could take the learners outside to where there is a flat, smooth surface. You could use the corridor, the school hall or any other flat area which is convenient for you. Collect a variety of different sized balls (or spherical objects), boxes (or prism objects) and cylinders, and take these out with you. It would be a good idea to get learners to work in groups so that they can compare their findings. Try to keep the groups to a manageable size, so that all learners can see and participate in the activity. You can then give each group a collection of different objects from the ones that you collected and took outside with you.

Questions are extremely important as they encourage verbalisation and the development of learners understanding. Some possible questions include:

“Which of the objects do you think you can roll?”

“What are these objects called?”

“Why do spheres/ball shapes roll? (Because they have curved/round surfaces.)

“Why do box shapes slide?”

“Why do cylinders roll and slide?”

You can use your questions to help learners realise that some objects will roll, some objects will slide, and some objects will both slide and roll.

Transformations

We are now going to look at the idea of transformations. Transformations occur when a shape changes its position, its size or its shape. Transformations that do not change the size or shape of an object are called rigid motions. We will focus on the three rigid motions of translations, reflections and rotations.

Translations are when an object or shape ‘slides’ into a new position. When a shape has been translated, every point in the shape is moved the same direction and distance. You can involve the learners in activities by getting them to pull or push shapes ALONG a line, BELOW or ABOVE a line in order to draw translations. The shapes in the grid below have been translated. The orientation of the shape stays the same.
Rotate means TURN like you turn a door handle when you open a door. We can turn ANY object around, in a variety of ways: in space, in the plane, about a point (inside the shape), or about a point (outside of the shape). We can also turn the shape through any number of degrees. The rectangle below has been rotated through a few different degrees. You can see this because you see the same rectangle but lying at different angles. The orientation of the shape changes.

A reflection is a type of rigid motion in which the object or shape is flipped to create the ‘mirror image’ of the original shape. This transformation acts like a mirror, where all the points on the original shape are exactly reflected on the opposite sides of a line. The line is called the line of reflection. The reflected image has the same size and shape as the pre-image, but it faces the opposite direction.

The first and last image below show examples of reflections. The heart has been rotated and the semicircle has been reflected and translated.

**ACTIVITY 3**

- Why do you need to know more than just the information required to teach learners a particular content area?
- What is the value of playing and building with, and drawing 3-D objects?
- Describe how physical activity with shapes will help learners to develop their understanding of transformations.

**Commentary**

As teachers of young learners, it is easy to give in to the temptation of just knowing the information necessary to teach the learners in your class. Many teachers do not feel confident in their own understanding of Maths, and so they just know enough to get through each lesson that they teach. Unfortunately, this strategy doesn't allow for the possibility of questions or misconceptions that arise which then can't be addressed by the teacher. It is essential that teachers know more than they need to teach the set curriculum. Teachers’ own understanding should go beyond what they are explaining to learners, as their extended knowledge will help them to develop the learners understanding and reasoning skills.

Luckily, by working with concrete apparatus and being physically involved with activities, it is possible to extend our own knowledge and understanding of Maths as well as that of the learners. For many adults, we were taught maths by just completing problems and writing in workbooks. We may not have had much opportunity to work with concrete apparatus ourselves, which could have resulted in a limited knowledge base and reduced confidence. As we plan and prepare activities for the learners, where they can play and build with shapes and objects, we provide opportunities for the consolidation of understanding by physically seeing and experiencing each of the concepts being addressed.
Check your understanding: Multiple Choice

1) Which of these is not one of the dimensions in a 3-D shape?
   A) Height
   B) Volume
   C) Width

2) The word used to indicate where two surfaces meet is:
   A) Face
   B) Vertex
   C) Edge

3) Building towers is a worthwhile learning experience because:
   A) Learners learn more about the properties of 3-D objects.
   B) Construction is good occupation.
   C) The activity helps learners focus and concentrate.

4) A translation is a:
   A) Turn
   B) Flip
   C) Slide

REFLECTION

• Reflect on your experience of balls and boxes. Have you experimented with sliding and rolling?
• Think about when you could include this type of activity in your own teaching.

Well done you have completed Lesson 3.
In the lesson, we will discuss sorting and comparing activities, where learners can consolidate their understanding of the properties and features of 3-D shapes. We will then look at how to extend learners learn so that they can begin to move from Level 0 to Level 1. Most learners from Grade R - 3 will be working in Level 0, but it is important that you are prepared to challenge learners who are ready to move to Level 1. Finally, we will consider some ways to ensure that we are meeting the needs of all the learners in our classes.

What you will learn in this lesson

• Sorting and comparing 3-D shapes
• Moving towards van Hiele Level 1
• Teaching for all learners

Sorting and comparing 3-D shapes

When learning about 3-D shapes, learners need multiple opportunities to sort and compare concrete resources. They need to experience holding real objects so that they can feel the faces and edges. By doing this they will learn to make comparisons as they talk about the similarities and differences between the shapes. They can also spend time fitting shapes together to make bigger shapes (composing shapes) or breaking larger shapes down to make smaller shapes (decomposing shapes).

Some practical activities that you can use in your classroom include letting learners construct 3-D shapes by using salt dough to model the figures or using rolled up newspaper with masking tape. They could also use straws or coffee stirrers with twist ties or elastics to tie the ends together. It is important to make sure that you build up your activities from easy to medium difficulty, before moving on to more difficult tasks.

As learners develop their understanding, they can make a list of the properties of 3-D shapes that they have investigated. They can come up with their own definitions for shapes, and as they start to learn the correct terminology, they can begin to include this in their definitions. The learners can then use this knowledge and understanding as they participate in a variety of construction activities.
ACTIVITY 1

Watch the video "Faces of 3-D objects" (6:53 minutes) where the teacher works with a Grade 1 class, learning about the faces of 3-D objects.

• What do you notice about the involvement of the learners?
• How does this involvement help the learners to develop their understanding and to correct any misconceptions?

Commentary

In the video, the learners are all actively participating in the lesson. They all have concrete resources in front of them, that they are using these to discuss the faces of the shapes. This type of activity takes a bit of planning and preparation, as you will need to collect the boxes in advance. However, it is a worthwhile exercise, as the learners can physically see the properties and features of the shapes that they are learning about.

You will notice that one learner struggles to count the number of faces on his box, despite the fact that he is touching the sides as he counts. The teacher asks another learner to show him, which is a great idea. Learners often learn well from their peers and giving them the chance to talk and share their ideas is a valuable way of addressing misconceptions. When the learners are given the opportunity to draw around their boxes, they can easily see the shape of the face. This helps them to see the relationship between 3-D objects and 2-D shapes. This activity will also help them to describe their 3-D shapes, as they start to recognise that they can say that a 3-D shape (a cuboid) has 2 square faces and 4 rectangular faces.

Moving towards van Hiele Level 1

When learners have developed their confidence in their understanding of Level 0 concepts and activities, you can then begin to prepare them to move towards van Hiele Level 1. As we discussed in Lesson 1, Level 1 is the Visualisation level, where learners recognise and name shapes. As they move to Level 1, which is the Analysis level, they begin to explore the properties of shapes. In this level, learners need ample opportunities to draw, build, make, put together and take apart two-dimensional and three-dimensional shapes. They need to focus on the properties of the shapes, rather than simply identifying the shapes. Learners can be extended by challenging them to test their ideas about shapes by comparing a variety of shapes to see whether they all have the same features. Ask questions that get learners to think about whether the statements they make about the properties of a group of shapes apply to all the shapes in that group.

Remember that learners will not all be at the same level at the same time, so you need to prepare your activities so that you can meet the needs of all the learners in the class. You can structure your activities so that learners at all levels of understanding will have tasks that both provide opportunities for success as a certain amount of challenge. With careful observation, it is possible to assess at which levels the learners are working. Listen carefully to their comments and observe their reasoning, as these will give you a good indication of their level of understanding.
ACTIVITY 2

Watch the video "3-D objects" (4:12 minutes) where the Grade 3 learners ask questions to find out which 3-D shapes are in the teacher's bag.

What terminology can you hear being used in the activity?

- Do you think this is a good way for learners to learn about the features of 3-D shapes?
- Give reasons for your answer.
- How could you extend this activity?

Commentary

In the video, the teacher is encouraging the learners to use the correct terminology to describe the shapes (as we identified in Lesson 3). The learners are likely to remember this activity, and they will be able to use the language confidently in follow up lessons, as they are enjoying the game-like activity. This activity is more playful than a worksheet or written task would be, and so the learners are actively engaged in trying to work out the correct shape.

The teacher could extend this activity by getting the learners to compare 3-D objects. She could hold up a cylinder and a cone and ask, “What is the same about these objects?” Learners would need to note that both shapes have flat and curved surfaces. She could then ask, “What is different?” The learners would have to note that the cylinder has two flat surfaces, but the cone has one flat surface. The cone also has one pointy end, whereas both ends of the cylinder are flat. The teacher could then do the same with other 3-D objects, comparing them according to their characteristics.

- Cube and rectangular prism;
- Cone and the pyramid;
- Cone and sphere;
- Prism/cube and pyramid;
- Sphere and cube.

Teaching for all learners

In South African classrooms, we are likely to have learners with a variety of different needs. These needs may be learning needs, language needs, or even social, emotional or economic needs. In order for learners to learn effectively, we need to identify these needs and try to accommodate them as best as we can in the classroom. The first thing we need to establish is what the learners have as their existing, or prior, knowledge. The knowledge that learners bring into a lesson influences their ability to take in new information. We need to create an environment that focuses on learning with understanding. This type of learning is based on connecting and organising knowledge around the main conceptual ideas. This means that we can start with what
learners already know, and then build onto this knowledge, helping them to find patterns and relationships between ideas in order to simplify their learning.

It is essential that you take the context into consideration. Think about what is relevant for your learners, and perhaps make modifications to the content or curriculum so that you can better meet the needs of the learners. However, remember that this does not mean that you can have lower expectations of your learners. You need to maintain high expectations, and challenge learners to be resilient as they learn new information.

In a learning-centred mathematics classroom (spoken about in the mathematics framework for teaching mathematics in South Africa, DBE, 2018), you need to provide clear explanations of concepts and procedures in order to address learners’ errors and misconceptions. Learners should verbalise their mathematical ideas, and work towards making connections between topics. In order to do this, you should select and design tasks that emphasise key mathematical ideas and encourage learners to actively participate in each activity.

ACTIVITY 3
Think about the learners in your class.
• What different needs can you identify in your class?
• What are you currently doing to meet the different needs of learners in your class?
• What else could you do to meet these needs more effectively?

Commentary
When you establish a learning-centred classroom (DBE, 2018), there are four main areas on which teaching and learning activities should be built. These are conceptual understanding, procedural fluency, strategic competence and reasoning.

In order to develop conceptual understanding, teachers need to design lessons and activities that provide opportunities for learners to grasp mathematical concepts, operations, and relationships. In the context of our space and shape topic, this specifically refers to the development of learners’ ability to learn key concepts in relation to 3-D objects, 2-D shapes and their properties.

When learners are procedurally fluent, it means that they are able to carry out procedures flexibly, accurately, efficiently, and appropriately. When learning about shapes, the learners should develop procedural fluency in their ability to recognise, identify and name 3-D objects.

Strategic competence refers to the ability to identify and use appropriate strategies to solve mathematical problems. As learners participate in activities that involve building with 3-D objects and making 3-D objects using 2-D shapes, they will consolidate their ability to recognise the properties of 3-D objects.

Teachers need to provide many opportunities for learners to develop their mathematical reasoning skills. This is an essential skill that includes logical thought, and the ability to reflect, explain and justify. In our topic on Space and shape, learners will be able to justify and explain the relationships between objects using the properties of 3-D objects.
Check your understanding: True or false?

1) Decomposing shapes means that you break down larger shapes in order to make smaller shapes. **True**

2) In Level 1, learners need to focus on identifying the shapes rather than on the properties of the shapes. **False** - They need to focus on the properties of the shapes, rather than simply identifying them.

3) Your context means that you can lower your expectations to meet the needs of the learners in your class. **False** – You need to maintain high expectations, and challenge learners to be resilient as they learn new information.

4) Developing reasoning skills is part of a learning-centred classroom. **True**

**REFLECTION**

- Think about your learning environment. Do you think it is representative of a learning centred classroom?

- Write a personal goal in relation to your engagement with the learners in terms of developing their participation in class.

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**Well done you have completed Lesson 4.**

**Answers:**

1. True
2. False - They need to focus on the properties of the shapes, rather than simply identifying the shapes.
3. False – You need to maintain high expectations, and challenge learners to be resilient as they learn new information.
4. True
In this lesson, we will explore the concept of symmetry. This can be a tricky concept for learners, and we need to be careful to make sure that learners fully understand it before giving them tasks that are too challenging for them. We will then look at the idea of position, where learners learn to identify where a shape is in space. Language is particularly important here, as we then move on to direction, where learners will need to describe how to move to an object using the correct terminology.

What you will learn in this lesson
- Symmetry
- Position
- Direction

Symmetry
Symmetry is division of a 2-D or a 3-D shape into identical parts. When we say symmetry, we mean line symmetry. Line symmetry is also called reflection symmetry (because it has a lot to do with reflections) and bilateral symmetry (because of the ‘two-sided’ nature of symmetrical figures). We say line symmetry because of the line of symmetry in 2-D shapes - the line (or axis) about which the symmetry occurs. When two points are symmetrical to each other we say that the one is the reflection of the other. Symmetry can be horizontal, vertical or diagonal, and a shape needs to have only ONE axis of symmetry to be symmetrical, though it may have MORE THAN ONE axis of symmetry.

The topic of symmetry lends itself well to practical activities in the classroom. You could use any of the following:
1. Folding and making holes in paper with a compass/pen nib.
2. Folding and cutting paper shapes. Experiment with one and more folds and cutting on the different edges, too. Let the learners predict what the shape will look like before they open it up.
3. Point plotting on a coordinate grid or working out the unknown co-ordinates of a given symmetrical shape.

4. Using mirrors with real objects (e.g. pencils, sharpeners) and with drawings (familiar or unfamiliar).

5. Paint blobs on one side of a piece of paper and then squash two sides of the paper together along a fold. Learners love to see what interesting symmetrical images they can produce.

6. Colour blocks in a grid in a symmetrical pattern.

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**ACTIVITY 1**

Watch the video "Symmetry (3)" (4:57 minutes) where the Grade 3 learners learn about symmetry by colouring in blocks on a grid.

- What terminology do you hear in this activity?
- How are the learners involved in the activity?
- What do you notice about the way that the teacher addresses the one learner’s error?
Commentary

Hands-on activities are best as they involve the learners, and give them an opportunity to construct their own understanding. The teacher used the words horizontal and vertical, and demonstrated with her hands to help the learners remember what these words mean. The learners need multiple opportunities to:

- identify shapes that are either symmetrical or not symmetrical;
- insert line/s of symmetry in given drawings;
- complete drawings to make them symmetrical; and
- create symmetrical designs

In the video, the teacher helped the learners to recognise that some of the blocks had been coloured in incorrectly. She encouraged the learner to realise which blocks needed to be coloured in instead, rather than just giving the correct answer herself. If learners are struggling to identify the symmetrical picture, then it would be a good idea to use paper folding or a mirror to help them. These activities make it a bit more obvious what should be on the other side of a line of symmetry, and learners find it easier than just looking at a flat drawing.

There is symmetry all around us, in our homes, in nature and in art. Ndebele art is particularly interesting to study since it is full of geometrical shapes, symmetry and other transformations. Learners could be encouraged to draw their own Ndebele artworks as shown in the pictures below, linking their learning about maths to cultural knowledge.

Position

One of the goals of the Space and shape topic is for learners to understand the concept of location. They need to be able to describe the position of a shape in space. They can do this by describing the shape's position in relation to that of another shape, or by considering the shape's position on a grid.

Learners are comfortable describing position or location in their everyday, social language. They do this all the time in conversation. However, sometimes this social language is not clear or specific enough. Learners may be inclined to say “The square is over there” which doesn’t actually give us any real indication of the location of the square. Learners need to be given opportunities to develop their vocabulary of position. One way that they can do this is by creating maps of familiar objects, where they can indicate the position of an item by referring to its proximity to another item. It is worthwhile to allow learners to construct these maps in a concrete way. They can use real items or multifix blocks to do this. The concrete items help learners to better develop their understanding.

Learners find bird's eye view maps extremely difficult, and they will develop misconceptions if this is introduced too early. Concrete maps form a solid foundation on which learners begin to understand that pictures and symbols can be used to represent the real objects. Once this foundation has been established, learners can move on to learning about movement on a grid. They will have objects scattered on a grid, and they will learn to use the correct terminology to describe their movement across the grid to a specified object. Once learners have grasped the idea of location, they can then investigate the movement of shapes.
ACTIVITY 2

Watch the video “Position and direction” (2:38 minutes) where a Grade 2 class uses the correct terminology to describe the position of objects.

- Make a list of the position-related vocabulary that you hear.
- What could you do to get the learners to use the vocabulary more themselves?

Commentary

In this video, the teacher helps the learners to see that they can describe the position of objects by using position-related words. It is important to use concrete objects, so that the learners can move the objects around themselves. By doing this, they get the opportunity to see the differences in the position of the object, and to construct their own understanding of the new terminology.

It is a good idea to get learners to use this vocabulary regularly, rather than in just isolated lessons. You could use the time in between lessons, or when you notice learners are getting a bit tired and need a short break, to call out some instructions using the terminology. You could say, “Stand on the left of your chair. Put your pencil under your table. Put your book on top of the pencil”. A few short instructions like this could work well as you change from one lesson to next, giving the learners a ‘brain break’ as well as an opportunity to practice the new language.

Direction

We have discussed the idea of position in maths, which is the ability to describe where one object is in relation to another object. In the video you watched for Activity 2, you would have noticed the teacher getting the learners to follow directions in order to move around the classroom. In Maths, direction involves describing the way in which we need to move, for example forwards, backwards, left or right. We also use our knowledge of fractions when working with directions. Learners will need to be able to turn a quarter turn or a half turn.
Learners progress from describing the position of objects and shapes in patterns, to understanding how to explain movement in a straight line, as well as rotations. They will use the terminology “clockwise” (turning in the direction of the hands of a clock) and “anticlockwise” (turning in the opposite direction of the hands of a clock), as well as “left” and “right”.

Once again, it is important to get the learners to physically move around themselves. By having to follow the directions, and actually move their bodies according to the instructions, they will better understand both the terminology as well as the necessity for clear directions. Learners enjoy working in pairs, where one learner gives the directions and the other learner has to follow the instructions in order to reach a designated point. If you have access to an open space where learners could move around safely, you may even challenge the learners further by blindfolding the learner following instructions. By doing this, the learner giving instructions needs to be very specific and clear, as the blindfolded learner cannot look where they are going in order to help find the way.

**ACTIVITY 3**

Look at the coordinate grid below. The $\times$ is positioned at (3,2) and the $\circ$ is at (1,3).

![Coordinate Grid](image)

- Ask a friend to play Noughts and Crosses with you on the grid.
- The first person to get three in a row is the winner.

**Commentary**

Using a game to teach learners about location is a great way to actively involve them in the construction of their understanding. In the example in Activity 3, the learners would learn how to identify coordinates by referring to the X axis (the horizontal line) first, and then the Y axis (the vertical line). The coordinates are written in brackets, separated by a comma. In the early grades you could use a grid to help learners to determine the position of items. Learners will use the terminology to describe how they would move across the grid to get to a specified point on the grid. For example, to get to the airport you would need to move **forward** two places from the starting block. You would then need to move one block to the **right**.
### Check your understanding: Multiple Choice

1) **Shapes can have:**
   - A) One line of symmetry
   - B) Three lines of symmetry
   - C) Multiple lines of symmetry

2) **Which activity would be a good way to teach symmetry?**
   - A) Paper folding
   - B) Using a mirror
   - C) Both A and B

3) **Learners can develop their understanding of position by:**
   - A) Creating concrete maps using physical resources.
   - B) Writing sentences with the new terminology.
   - C) Listening carefully to the teacher.

4) **When learners learn about direction, a key factor is:**
   - A) Practice
   - B) Vocabulary
   - C) Context

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### REFLECTION

- Think about the way you learned about symmetry at school? Would you teach it in the same way or differently and why?
- How do you think that you can help learners to learn about symmetry effectively?

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**Well done you have completed Lesson 5.**

**Answers:**
- 1. C
- 2. C
- 3. A
- 4. B
The topic of measurement (or size) is one in which we teach our learners how to measure. We need to ensure that our learners understand the measuring process fully. To do so, the first need to understand the measurement concepts (of for example length, mass, capacity and volume) before we begin to teach them the skills of using measuring instruments. Simply put - you need to know what you are measuring before you can measure it. In this lesson you are introduced to ideas across the continuum of the teaching of measurement: starting with the foundational concepts of measurement, the use of units of measurement and problem solving in measurement contexts. These ideas relate to the teaching of measurement across all grades. You need to understand and be able to apply all of them yourselves in order to teach the topic well.

What you will learn in this lesson

• Measurement characteristics or attributes
• The relativity of size
• Standard units and non-standard units
• Word problems in measurement contexts - mass

Measurement attributes

Copley (2010) makes the point that adults often think of measurement in terms of formulas, rulers, and other measuring tools while young learners encounter measurement in many contexts every day as they explore and try to make sense of their world. Measurement involves quantifying the physical characteristics or attributes of physical objects that have size or amount. When we quantify, we assign a numeric value to something. For example, we can say that a belt is 90 cm long, or a cup holds 250 ml of water, or the mass of the learner is 34 kg. We cannot quantify things to which we cannot assign a numeric value. For example, if the belt is black, we cannot say how black it is by giving a number.

Learners are aware of physical objects and their characteristics before they develop a concept of number and measurement. We must ensure that they fully understand the concepts (of length, mass, volume or capacity for instance) of the things that we measure before we teach them how these are measured. This is because the way in which we assign numeric values to quantities is by comparing them to other quantities similar to themselves.

The relativity of size

Things which are absolute cannot be measured in degrees. They are, or they are not. They stand as they are. Size is not absolute. Size is relative and it is arrived at by comparison. We could say that something is long. What does this mean? How “long” is long? On the other hand, what is short? Perspectives differ, and different answers to these questions exist. That is what we mean by “size is relative” – it is given in relation to something else. It is this property of size that we use to quantify things. We compare them to “standard units” of themselves. Relativity of size is an idea we need to communicate to learners, even if in an intuitive way, without referring to relatives and absolutes!
We measure physical quantities not physical objects

What do we measure? We measure the size (amount) of an attribute of a physical object. Length, for instance, may be great or small. We measure the length of an edge, not the edge itself. We can say the length of the edge of a table is 56 cm. (We do not say the edge is 56 cm.) We may measure the mass of a ball. Then we would say the mass of the ball is equal to 3 kg. (We do not say the ball equals 3 kg.)

This shows the need for careful use of language in this topic of measurement, so that we avoid speaking unclearly or ambiguously. We must say exactly what we mean, and give clear instructions to our learners, so that they will know to which attributes we are referring. We must not allow any confusion between a thing itself (a table) and its attributes (the edge of a table).

ACTIVITY 1

• Think about the learners in your class. Which of their physical characteristics can be measured and which cannot?
• Write out a few clear instructions to learners, calling on them to measure some different physical quantities:
  ~ Relating to a learner
  ~ Relating to a desk

Commentary

The learners in your class have physical characteristics such as eye colour, hair, a smile, and personality; they are made of substance which is attracted by gravity; they have academic ability, artistic ability, sporting ability, shoe size; they have height, take up space, and so on. Some of these characteristics have size or amount – we call these physical quantities: mass, weight, length, volume – these are the attributes that we can measure.

• How tall are they? (length)
• How much substance are they made of? (mass)
• How much space do they take up? (volume)

When we ask learners to measure things, we need to use unambiguous, clear language. We would not say: measure that boy – we would specify which of the boy's attributes should be measured. For example we could ask them to measure the height or mass of the boy. If we want to ask for measurements of a desk, we do not say measure that desk we should ask them to measure the height or width or length of the desk. Each of those has a different measurement and learners need to know exactly what should be measured each time. Hands-on activities comparing physical quantities introduce learners to the idea of a unit of measurement, as seen in the pictures below.
Standard units and non-standard units

We choose suitable units to measure with. The units must possess the property of that which we are trying to measure. For example, to measure the length of the edge of a desk we could use a pencil because it has length, or to measure the length of the sides of a square we could use blocks. Then the block is 1 unit and the length of the side of the square is 5 blocks (units) as shown in the first picture below. It would be problematic if non-standard units were used to measure formally, since they are arbitrary and people could choose so many different non-standard units. As seen in the second picture below, if the blue blocks are used to measure the length of the car it is 3 blocks (units) long but if the purple blocks are used it is 6 blocks (units) long.

There are certain accepted standard units used for measuring the attributes. In the drawing above a ruler is lined up against a paperclip and a pencil, and we can see that the paperclip is 4 cm long and the pencil is 15 cm long. Centimetres are one of the standard units that we teach when we teach about measurement. Learners need to learn the names of the units, what they are used to measure and how to use the instruments of measurement. But before they reach the stage of learning about standard units, learners need to learn about the measurement concepts and the vocabulary related to these concepts.

ACTIVITY 2

Comparing lengths

Watch the video “Comparing lengths” (4:26 minutes) of a teacher in a Grade 1 class.

- Describe the activity the teacher uses in this lesson.
- In what way does the teacher encourage the use of length vocabulary in the lesson?
- Did this lesson involve estimation?

Commentary

In this lesson the teacher uses pieces of string as a concrete aid to give learners the opportunity to compare lengths. She gives learners pieces of string to work with and asks them to sort them into order of length. Then she puts them on the board, sorted from longest to shortest and calls on various learners to come to the board to point out pieces of string according to various
comparisons she gives. She uses the terminology long, longer, longest and short, shorter, shortest all with reference to the string. In addition to learners participating in the general discussion with reference to the display on the board, you could also ask learners in pairs at their desks to compare lengths of string, talking to each other and circulating to see that they use the language correctly.

This activity did not involve estimation but it is very important when teaching about measurement to give opportunities for learners to estimate measurements – especially at the time when they are learning about the concepts themselves.

**Word problems in measurement contexts – mass**

Once learners have established the measurement concepts and know how to work with them they will be called on to do word problems that deal with measurement units in various contexts. This will apply their understanding of operations as well as their knowledge of the units of measurement. Word problems that enable learners to think about mathematics are another critical component of the teaching and learning of mathematics – in the context of measurement (or other contexts).

**ACTIVITY 3**

**Working with units of mass**

Watch the video “Working with units of mass” (5:30 minutes) in which a teacher sets word problems for her Grade 3 class.

- Describe the activity the teacher uses in this lesson.
- How does the teacher encourage learner talk in the lesson?
- How would you add to/change the lesson if you did it in your Grade 3 class?

**Commentary**

The teacher has set some word problems for the class which she asks them to do. She tells them about the questions and then gives them time to work out the solutions. She calls on learners to answer questions, probing their methods of solution. Assumed knowledge here is that of multiplication and division – the teacher could go into deeper explanations of the methods of solution and also set more problems. She could also ask learners to make up their own questions of a similar nature and try to solve those together.
Check your understanding: True or False?

1) Size is absolute.  
2) Getting learners to measure their table is a good activity in which to involve learners.  
3) A pencil is a good example of a non-standard unit of measurement.  
4) Word problems in context allow learners the opportunity to practice the operations while consolidating their understanding of measurement.

REFLECTION

- Think about your own practice of introducing the measurement concepts. Do you take adequate care to use correct language when asking learners to measure things?
- When you introduce the concept of length to learners do you give them a lot of opportunities to work with concrete manipulatives? Describe some of the activities you do.

Well done you have completed Lesson 6.

Answers:

1. False – size is relative.  
2. False – instructions need to be more specific. For example, learners need to measure the length of the table.  
3. True  
4. True
The topic of measurement should be taught using apparatus and practical exercises wherever possible. In the early grades, the conceptual understanding of the measurement attributes is established. Throughout this course we have emphasised the importance of teaching skills based on an understanding of concepts. We must remember this in our teaching of measurement and facilitate it by giving a good conceptual grounding followed by sufficient practical exercises. In this lesson we discuss further skills and concepts related to the activity of measuring. Your understanding of these concepts will help enrich your teaching of measurement.

### What you will learn in this lesson

- What is measuring?
- Pure numbers and denominate numbers
- Precision of measurement
- Direct and indirect measurement
- Capacity and volume - how are they related?

### What is measuring?

Measuring is the process whereby we assign a number to a physical quantity by comparing it with a standard physical quantity. In the previous lesson we discussed non-standard and standard units. In our teaching of measurement, we use non-standard units to assist the formation of the measurement concept we are teaching before we introduce the commonly accepted standard units applicable to that which is being measured.

### Pure numbers and denominate numbers

Pure numbers relate simply to the concept of number, of **how much**, without concerning themselves with **of what**. Cardinal number concept (the focus of Part 1 of this course) is about pure numbers. **Denominate numbers are a special category of numbers which specify what is being counted.** This is an important distinction for you to remember in the teaching of size, since all measurements are denominate numbers.

When we give measurements, we must state the unit of measurement.

For example, we measure mass in kilograms (kg). The scale on the right measures mass in kilograms.

Learners need to know how to read a scale like this - they should be able to show (by drawing in the arrow of the scale) that the bag of flour on the right has a mass of 5 kg.
Can a measurement be precise?

Another question we need to think about in the teaching of size relates to the precision of measurement. Because size is relative, a measurement will be as precise as the measuring instrument you are using allows you to be. If you use the same measuring instrument to measure two separate quantities, your measurements will have the same precision. The precision of measurements is also affected by the instruments we use to measure. It is important to choose an appropriate unit of measurement since this affects the precision of a measurement. For example, we would not use teaspoons to measure the capacity of a bucket – teaspoons are too small relative to the size of a bucket. But we could use jugs to measure the capacity of a bucket.

A measurement could be faulty if the instrument is faulty. Human error can also lead to faulty measurements: simple errors of carelessness or incorrect reading of the instrument. Exact measurements are not possible, but a level of accuracy can be chosen which is appropriate to a situation.

ACTIVITY 1

• Why are all measurements given as denominate numbers? What does this mean for the teaching of measurement?

• Why do we say that no measurement can be exact?

Commentary

Measurements are always given as a number of units, whether non-standard or standard units have been used. You must teach your students to record their measurements correctly, giving the unit of measurement each time. Denominate numbers (units) need to be properly used when it comes to computations involving these numbers, once the concepts and measuring skills have been taught.

Measurements cannot be exact since there is always room for some error in the reading of a measurement – but we do try to measure as accurately as possible and we should teach our learners to do so as well. They need to know how to use the measuring instruments correctly and to record their readings correctly, using the unit of measurement.

Direct and indirect measurement

Certain quantities can be measured directly. Think about the length of a pencil or lengths of the edges of a square. A ruler can be used. Think about the volume of a cube. Unit cubes can be used. Most measurement activities in the early grades involve direct measurement.

But some measurement cannot be made directly. For example – how could you measure the perimeter of a piece of paper which has been cut into an irregularly shaped region, or the volume of an irregularly shaped stone? They cannot be measured directly, but both of these measurements can be found by using a procedure we call indirect measurement. This requires a certain level of abstraction and a clear understanding of the concepts involved if it is to be grasped.

Let us look at the example of the perimeter of an irregular region. Remember that perimeter is the measurement of the outside boundary of a shape. To measure an irregular perimeter we can use a piece of string. We take the string and lay it carefully all along the border of the shape.
Then we take the string away from the shape, cut it off so that we have the piece that will give us the measurement of the perimeter, straighten it up, and measure how long the perimeter of the shape is using the string and a ruler.

This is indirect because we did not place the ruler along the border of the shape, because we could not. We used an indirect method (string) which successfully enabled us to find out the perimeter of the shape.

To measure the volume of a stone we can submerge it in water and find out how much water it displaces. The amount of water displaced will equal the volume of the submerged stone.

ACTIVITY 2
• When you get into a bath, what happens to the level of the water in the bath?
  o Why do you think this happens?
  o What does this show you?

Commentary
This is another example of indirect measurement. If you step into a bath, you displace the water in the bath. The water level rises. This shows that your body has volume! It takes up space.

Volume and capacity
Volume is the measure of the amount of space an object takes up. Capacity is the measure of the amount of space inside, or the ability of an item to hold something. Everything that takes up space has a volume. Containers that are hollow and can hold things (such as liquids) are said to have capacity. When a container is full, we say it is filled to capacity. That means it is holding as much as it is able to hold. Volume and capacity are related. The liquid that you pour into a container has volume - the volume of water that it takes to fill a container is equal to the capacity of the container. Because of this relationship, volume and capacity can both be measured in millilitres, but volume is also measured in cubic units.

Solid objects also have volume - for example the cube illustrated above has a volume of 25 cubes. Your body has volume - as discussed in Activity 2 above - if you step into a bath your body displaced an amount of water which is equal to the volume of your body that has gone under the water.
ACTIVITY 3

Comparing volume and capacity

Watch the video "Comparing volume and capacity" (4:27 minutes) of a teacher in a Grade 1 class teaching about capacity.

• Describe the activity the teacher uses in this lesson.
• What other questions might you ask if you used this activity in your class?

Commentary

In this activity the teacher asks her class to sort a set of four containers according to capacity. The learner that comes to the front sorts them from the tallest to the shortest container and evidently believes that the tallest one will hold the most and the shortest one will hold the least. The teacher has filled the first container (a 1 litre coke bottle) with water and proceeds to ask the learner to pour the water from one container to the next. They see that these containers all have the same capacity – 1 litre. This is a nice activity to show learners not to be deceived by the height of a shape when thinking about the capacity of the shape. You could ask questions that call on the learners to think more about heights of the containers and their volumes. Remember to ask questions in such a way that learners are made to use the measurement vocabulary themselves and not just answer simple yes/no questions.

When you teach capacity you can easily collect empty containers and bring them to class to use in demonstrations, or even better, bring enough for all learners in the class to work in groups doing hands-on activities. In the activity suggested by the series of pictures below, the teacher works with her class to compare the capacity of some different containers using a glass. See how she made a funnel using a cut off bottle and involves the learners in the demonstration.
### Check your understanding: Multiple Choice

1) **Measuring is the process whereby we:**
   - A) assign a number to a physical quantity by comparing it with a standard physical quantity with the same attribute.
   - B) assign a number to a physical quantity by comparing it to a different physical quantity.
   - C) Estimate the size of a physical quantity by comparing it to another physical quantity.

2) **A measurement is:**
   - A) Perfectly accurate.
   - B) As precise as the measuring instrument allows.
   - C) Different depending on the time and place of the measurement.

3) **An indirect measurement is:**
   - A) A measurement completed by someone else.
   - B) A measurement recorded in a book.
   - C) A measurement completed by using a piece of string and a ruler.

4) **Capacity is:**
   - A) the measure of the amount of space an object takes up.
   - B) the measure of the amount of space inside an object.
   - C) The measure of the distance an object covers.

### REFLECTION

- Reflect on your understanding of the relationship between volume and capacity. What do you think is the best way to make this clear to young learners.
- Write a personal goal in relation to your teaching of measurement – in what way will you deepen your own understanding of measurement concepts so that you will be able to teach them with greater insight.

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**Well done you have completed Lesson 7.**
Conservation tests give us a way to establish whether or not a learner has understood a certain measurement concept. In the next section we look at some introductory exercises that can be used in the establishment of the size concepts.

What you will learn in this lesson

- Conservation tests for the teaching of measurement – length and area
- Conservation tests for the teaching of measurement – mass
- Measurement of time

Piaget’s conservation tests

The ideas of the developmental psychologist Piaget about conservation of number are well known. In this lesson we use his ideas again, to check our learners’ readiness to proceed with the measurement of things such as length, mass, area, volume and capacity. As with number concept, we need to check that learners have achieved conservation of these concepts before we can teach about their measurement. Conservation of the concept means that they have a clear understanding of the constancy or unchanging nature of length, mass, area, volume and capacity. In other words, they understand the meaning of these concepts.

In this lesson we will look at conservation tests for each of length, mass area, volume and capacity. The tests follow a similar pattern but vary according to the measurement concept involved. For example, before we look at each test separately, we say that a learner has achieved conservation of length once they are aware that the length of a piece of string remains the same, no matter if we lay it straight, curve it, roll it up or even cut it up. So the conservation tests are all designed to check whether the learners know that equal amounts remain equal even when their appearances have been distorted.

Learners develop at different paces, and we cannot assume that they will all achieve conservation of the measurement concepts at the same time. It does not take very long to test for the conservation of these concepts, so we should always just take that little extra step to check for conservation before we proceed to teach how to about measurement and the use of units of measurement.

Piaget went further to say that if a learner is able to explain that the distorted amounts could be restored to their original appearance, then the learner has achieved the concept of reversibility, another affirmation of the understanding of the concept.

ACTIVITY 1

- Why are the conservation tests useful in the teaching of measurement?
- Give an example of what you think is meant by reversibility in relation to the concept of length.

Commentary

The conservation tests are useful because they help a teacher find out whether or not a learner has understood a concept. They are an effective way to test for understanding since the answers
learners give are the evidence of their understanding or lack of understanding. If a learner has achieved reversibility in relation to the concept of length it means that they know that a piece of string (for example) has been cut into pieces, the total length of the individual pieces will be same as the total length of the original piece of string.

**Conservation of length**

**Length is the measurement of the size (how long?) of an edge (straight or curved).** In the early grades, learners develop their understanding of the concept of length by comparing objects directly (for example by lining up two objects side by side to see which is longer). To check for conservation of length, show the learner two pieces of string that are the same length. Let her satisfy herself that they are the same length.

Now take one of the pieces of string and twist it around into a coil. Ask the learner if the two pieces of string are same length, or if their lengths are different (second display). You could then further distort the one piece of string by cutting it up into a few pieces (third display). Then ask again if the two displays contain the same length of string.

![Initial display](image1) ![Second display](image2) ![Third display](image3)

If the learner answers that the pieces of string are the same length, she has achieved conservation of length. If she answers no at any stage, then she is not sure that the length of the string remains the same even if its appearance is changed, and she has not achieved conservation of length.

If she can explain why they are still the same lengths in terms of restoring them to their original shapes, she has achieved reversibility of the concept of length.

**Conservation of area**

**Area is the amount of surface covered by a shape.** Learners develop their understanding of the concept of area by using a variety of materials to cover the surface of shapes. To test for conservation of area, show the learner two postcards which are exactly the same. They have the same area. Let her satisfy herself that they have the same area.

![Initial display](image4) ![Second display](image5)

Now take one of the postcards and cut it into two parts (second display). Ask the learner if the two areas covered are still the same, or if they cover different areas. You could then further distort the one postcard by cutting it up into a few pieces (third display). Then ask again if the two displays still cover the same area.
If he can explain why they still have the same area in terms of restoring them to their original shapes, she has achieved reversibility of the concept of area.

**Conservation of mass**

*Mass is the amount of matter of which an object is made*, although people often speak about the weight of an object, rather than its mass. Scientifically, weight is a measure of the pull or force of gravity on an object. Young learners do not need to know this distinction but it is better for teachers to model the correct terminology and speak about mass (which is consistent with the CAPS curriculum). Learners in the early grades develop their understanding of the concept of mass through comparison activities. They might use their hands as ‘balance scales’, holding an object in each hand to determine which object is heavier.

To test for conservation of mass, show the learner two balls of clay which have the same mass. Let her satisfy herself that they have the same mass.

Now take one of the balls of clay and roll it into a thin sausage (second display). Ask the learner if the two pieces of clay have the same mass, or if their masses are different. You could then further distort the one lump of clay by cutting it up into a few pieces (third display). Then ask again if the two displays contain the same mass of clay.
If the learner answers that the lumps of clay have the same mass, she has achieved conservation of mass. If she answers no at any stage, then she is not sure that the mass of the clay remains the same even if its appearance is changed, and she has not achieved conservation of mass.

If she can explain why they still have the same mass in terms of restoring them to their original shapes, she has achieved reversibility of the concept of mass.

**Conservation of volume**

*Volume is the amount of space taken up by an object.* In testing for conservation of volume you could use the same balls of clay that you used in the tests for conservation of mass. Show the learner two balls of clay which have the same mass, and which therefore have the same volume. Let her satisfy herself that they have the same volume.

**ACTIVITY 2**

- What steps would you now go through to test the learner for conservation of volume? Explain in writing and illustrate your demonstrations for the initial, second and third displays.
- How would you know whether or not the learner has achieved conservation of volume?
- How would you check whether or not the learner has achieved reversibility of the concept of volume?
- What other apparatus would be useful in tests for conservation of volume?

**Commentary**

You would go through the same steps and in the same manner as the conservation tests for length, area and mass were carried out – but in with a focus on the volume of an object. Similarly, if the learner can answer that the volume does not change even if it has been distorted, he has achieved conservation of volume. If he can explain why they still have the same volume in terms of restoring them to their original shapes, she has achieved reversibility of the concept of volume.

In lesson 7 you watched a video of a lesson where learners established the fact that a group of containers (of different shapes) have the same capacity. This activity draws on learners understanding of the conservation of volume (that an amount of water, if poured from one container to another, even if it looks different, remains the same). This hands-on activity thus helps learners establish the capacity of the containers using volume. Learners might believe that the containers hold different amounts (volume) of water. They test this using water (a fixed volume), poured from one container to the next to discover/prove that the containers all hold the same amount – in other words, have the same capacity.
Measurement of time

We speak about time so often in our daily lives. Time is something that we measure but we cannot see it or touch it! Yet we are aware of the passing of time and we do measure it. Time is the duration of an event from its beginning to its end. Regular teaching about time is needed because time is abstract – we cannot touch it or feel it, but we are aware of its passing. Learners become aware of time early in their lives because of the regular cycles of night and day, the seasons and the years. Teachers should help learners estimate, measure, and describe the way time passes. Learners need to learn both how to tell the time (using clocks) and also how to calculate how much time has passed. Introductory activities on the teaching of time link to sequencing of events which help learners to think about the passing of time. Then learners need to learn about the units of time, of which there are many – since time is measured years, months, weeks days, hours minutes and seconds – and more!

ACTIVITY 3

Hours and half hours

Watch the video “Hours and half hours” (3:10 minutes) of a teacher teaching a Grade 2 class about time and telling the time. The teacher recaps learners’ experiences of time and the importance of being able to tell the time.

• What is the value of recapping learners’ experience of telling the time?
• What sort of clock does the teacher use in her lesson? What is your experience of learners working with analogue clocks?
• This lesson is about hours and half hours – to what other maths concept could this be linked?

Commentary

Awareness of time and the integral role it plays in our daily lives will motivate learners to develop their skills of telling the time and working with units of time. It will also help them to become more responsible. This teacher used an analogue clock with hands that moved correctly together – the value of this is that it is realistic and can help learners see how the hands move together. Many learners do not know how to tell the time using an analogue clock because they are more used to digital clock faces. Analogue clocks are still found and in maths teaching fraction concept (of halves and quarters for example) can be linked to the passing of time on an analogue clock.

The hour hand goes around the clock two times in one day.
12 hours and 12 hours is 24 hours.
The minute hand goes around the clock every hour.
There are 60 minutes in an hour.
30 is half of 60. When the minute hand points to the 6, we say half past.
Check your understanding: True or False?

1) Conservation refers to a clear understanding of the unchanging nature of length, mass, area, volume and capacity.

2) Area tells us how long the edge of shape is.

3) Weight is a measure of how heavy an object is.

4) Time is only measured in hours and minutes.

REFLECTION

• Reflect on the value of the conservation tests and how you could use them when you teach measurement concepts.

• What apparatus would you use to test for conservation of length, mass, capacity and volume?

Well done you have completed Lesson 8.
Data handling is the topic in the curriculum that touches on statistical concepts in the early grades. In this topic we start to develop, represent and interpret statistical information. If, for example, someone says to you, 55% of the learners at their school are boys, while only 5% of the teaching staff are males, they have given you some statistical information. This might make you think about the ratio of boys to male staff. Statistics help us to summarise information and make comparisons. Statistical results are often shown as graphs as these visual representations show off the statistics more effectively than words or tables might do it. The graphs are meant to make the interpretation and analysis of the information represented easier.

In the first of the two lessons on data handling in this course, we will outline the data handling cycle and some of the terminology which are used in data handling. We will then think about why we should teach Data Handling in the early grades.

**What you will learn in this lesson**

- The data handling cycle
- Data handling vocabulary
- Why should we teach Data Handling in the early grades?

**The data handling cycle**

- **Collect and organise data**
  - Collect and sort everyday physical objects.
  - Collect data about the class or school to answer questions posed by the teacher.
  - Organise data supplied by teacher or workbook/textbook
  - Organise data in
    - lists
    - tally marks
    - tables

- **Represent data**
  - Draw a picture of collected objects.
  - Represent data in
    - pictograph (limited to pictographs with one-to-one correspondence)
    - bar graphs

- **Analyse and interpret data**
  - Answer questions about data presented in
    - pictographs (limited to pictographs with one-to-one correspondence)
    - bar graphs

- **Discuss and report on sorted collection of objects**
  - Give reasons for how collection was sorted;
  - Answer questions about
    - how the sorting was done (process)
    - what the sorted collection looks like (product)
  - Describe the collection and/drawing
  - Explain how the collection was sorted
ACTIVITY 1

Representing and interpreting data

Watch the video “Representing and interpreting data” (3:55 minutes) of a teacher working with a pictograph in a Grade 1 class.

• Describe the activity the teacher uses in this lesson.
• Which parts of the Data handling cycle are covered in this lesson?
• If the lesson were extended could other parts of the cycle be included? Which of them and how?

Commentary

In the video, the teacher used a pictograph to discuss the favourite colours of the learners at the school. The colours are represented by blocks of colour, and the teacher explains that one block corresponds to one learner’s vote. We can see that in this activity, data is represented in a pictograph with one-to-one correspondence. This means that the activity covers the ‘Represent data’ part of the Data Handling cycle. In addition to this, the learners answer questions about data presented in the pictograph. This means that the activity also covers the ‘Analyse and interpret’ part of the cycle.

In order to cover the other two parts of the data handling cycle, we would have needed to begin the lesson by collecting the data ourselves. The teacher could have asked questions, and the learners could have recorded the information about their friends’ favourite colours. Once the information had been collected, then the learners would have needed to organise it in a logical way so that it was easy to read. This would have covered the ‘Collect and organise data’ part of the cycle.

We could then have moved on to the ‘Discuss and report on sorted collection of objects’ part of the cycle. This would involve getting learners to provide reasons for the way in which they organised the data. Learners could describe the data collection and explain how they have organised the information. This would be similar to the way in which the teacher explains the pictograph to the learners in the video, but there would need to be greater involvement from the learners.
Data handling vocabulary

There is a lot of vocabulary related to data handling. You need to be sure that you know the meaning of the terms described below so that you can use them yourself and apply them when you teach data handling lessons. Examples are given for each term, followed by an activity for you to work through to apply your understanding of each of the terms.

**Data** is information that is collected relating to a given topic. For example, you might want to find out information about the birthdays of the learners in your class. When you collect the information about birthdays, you are **collecting data**.

**Raw data** is data which has been collected but not yet sorted out in any way. For example, when you collect the information about learners' birthdays you could use a class list to do so. You could write the date of each learner next to their name on the class list. When you have finished collecting this data, it will not be easy to tell, from just looking at the list, if more birthdays fall in January than in July, since you have not yet counted up the number of birthdays which fall in each month. Raw data needs to be sorted.

A very common method used to sort data is to **tally** the data. You could use tallying to sort the data in the raw data example given above. To do so, you would write a list of the days (Monday to Sunday) on a page, and then go through the class list, making a mark next to the correct day for each name on the list. The tallies could then be counted up.

The totals that you get when you add your tallies give you the **frequencies** for the particular data collected. You could check that the total number of learners in the class is the same as the total you get if you add up all of the frequencies, to be sure that your tallying has been correct.

When we sort data into categories, the categories usually give us a way to **group** the data, which helps us to make sense of the data. For example, we could sort the birthday information we have collected. We could probably not sort this data according to date, as there would be too many different dates! There is very little chance that we would have many learners in one class with the exact same dates of birth, and so there would be almost as many categories as there are learners in the class. We could more effectively sort the dates we have collected according to days of the week (this would give us seven groups) or according to month (this would give us 12 groups). This would group the data in a more sensible manner and lead to meaningful comparisons.

**ACTIVITY 2**

A teacher gives her class an activity to find out about the colours of the cars that pass the school.

- Do you think this is an activity that can be completed easily?
- What are the benefits of this activity?
- What are the potential problems with this activity?

**Commentary**

When deciding on an activity which requires learners to collect data, you need to think about your context. For example, if you wanted to get the learners to find out about the colours of cars that pass the school, you would need to think about how many cars actually pass your school during the school day. If your school is on a quiet road where perhaps there are 4 cars that pass all day, then this activity would not be worthwhile. Equally, if the bulk of the traffic passing your school drove by at 7am, then this activity would not be very successful as you would probably not be able to get the learners to collect the data at that time of the morning. However, if your school is in a prime position to watch the cars passing (for a relatively short period of time) then this could be a valuable activity. Learners could collect the information themselves by observing the cars driving past. They could record the colours of the cars using tally marks, and then organise...
this information into a pictograph. Learners could also have an opportunity to interpret the information on their graph, and to communicate their findings to the rest of the class. This means that an activity such as this could allow learners to progress through all four parts of the Data Handling cycle.

**Why should we teach Data Handling in the early grades?**

Data handling is an important topic to cover in the early grades because it develops a variety of skills that can be applied to other areas of Mathematics and learning in general. While it can be a tricky topic for learners to grasp, it is still worthwhile because learners develop the ability to formulate and solve problems. As they collect, organise, describe and interpret data, learners learn to identify problems, and to find ways to resolve the problems independently of the teacher. This increased independence is a necessary step in their growth and development, and they will get further opportunities to develop in this way as they produce their own graphs. As learners learn to complete all the parts of the data handling cycle, they learn how to collect, organise and interpret information that they have sourced themselves. This teaches them to produce their own knowledge, rather than just waiting to consume knowledge provided by the teacher.

Another valuable aspect of the data handling topic is that learners learn that graphs are a means of communication. This ability to recognise that we do not just communicate through written and spoken words is an essential part of Maths. Learners need to understand that pictures, numbers and graphical representations provide an efficient method of communicating, and that they should be able to make sense of this information quickly and easily. By developing their confidence in interpreting information presented in different forms, learners will develop the ability to seek out information and answers for themselves, rather than just relying on information given to them by someone else.

**ACTIVITY 3**

- Do you think it is important to develop early grade learners’ independence?
- Give reasons for your answer.
- What could you do to develop learners’ independence when learning about graphs?

**Commentary**

Young learners can still be taught to be independent. It is imperative that we don’t under-estimate learners’ ability to think for themselves simply based on their age. Learners are capable of rising to the challenges set for them, and so we need to encourage them to think for themselves from a young age.

In the classroom, you can present the learners with appropriate problems that require data collection. Initially, it would be useful to give them guidelines for the data collection. You might point out to them some of the following things (depending on the problem which has been set):

- Data should be found using the correct sample. This means that the learners need collect information from the appropriate people.
- We cannot estimate, guess, or make up data. Actual data must be found.

If special instruments are needed in the recording of the data, ensure that the learners know how to use these instruments and check that the instruments are in good working order. This will help them work more independently. The learners might not know all of the graphical forms of representation, so you need to decide whether you will tell them which type of graph to use or if you will let them choose for themselves. Letting them choose for themselves is a great way to develop their independence, however you need to be sure that this meets your requirements in terms of the learning content.
### Check your understanding: Multiple Choice

1) Which of the following is **not** part of the Data Handling cycle?
   - A) Collecting data
   - B) Drawing data
   - C) Analysing data

2) Raw data is:
   - A) Irrelevant data
   - B) Incomplete data
   - C) Unorganised data

3) Frequency is:
   - A) The total that you get when you add your tallies.
   - B) How often you collect data.
   - C) The number of times you discuss your data.

4) Graphs are a way of:
   - A) Communicating information efficiently.
   - B) Hiding information.
   - C) Storing information.

### REFLECTION

- Why does the topic of Data handling lend itself to group work and projects?
- Write a personal goal in relation to making your teaching of Data handling more relevant to the personal contexts and lives of your learners.

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**Well done you have completed Lesson 9.**
In the second of the two lessons on data handling in this course, we will think about ways of collecting, sorting and representing information, this time in the context of tallies, pictographs and bar graphs. After this, the highest level of activity related to data handling in the early grades, interpretation of data, is considered. In this lesson we also discuss the ways in which Data Handling activities can be integrated into other topics in the curriculum, since Data Handling provides a context for the learning of other mathematical content.

What you will learn in this lesson

- Collecting, sorting and representing data – tallies and pictographs
- Collecting, sorting and representing data – tallies and bar graphs
- Interpreting data – pictographs and bar graphs

Collecting, sorting and representing data – tallies and pictographs

A pictograph is made up of little icons (pictures) which represent certain numbers of things as is indicated in the key, which must accompany the pictograph. The picture selected usually relates in some way to the data being represented. A pictograph looks and functions a bit like a bar graph, as you will see in the next section.

For a pictograph to show the number of cars passing through various tollgates on 24 September in a certain year, learners would be given a frequency table as shown below:

<table>
<thead>
<tr>
<th>TOLL GATE</th>
<th>NUMBER OF CARS PASSED THROUGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mooiriver</td>
<td>40 500</td>
</tr>
<tr>
<td>Grassmere</td>
<td>52 000</td>
</tr>
<tr>
<td>Kranskop</td>
<td>43 000</td>
</tr>
<tr>
<td>Middelburg</td>
<td>32 000</td>
</tr>
<tr>
<td>Kroonvaal</td>
<td>40 500</td>
</tr>
</tbody>
</table>

They could then use the data to create a pictograph to represent the number of cars passing through the tollgates:
ACTIVITY 1

Look at the picture below.

Could this picture be used in a data handling activity?
Which parts of the data handling cycle could be covered?

Commentary

This picture could easily be used in a data handling activity. It would work well because you could use it to cover all four of the parts of the data handling cycle.

To start off, the learners could collect the data from the picture by counting the number of circles, triangles, squares and rectangles in the picture. The learners would have to find a way to manage their data collection because they could get confused if they just count by pointing at the pictures. They may need to cross off shapes as they count them, or perhaps use tally marks to keep track of the number of shapes.

Once they have organised their data, learners can discuss and report on this data with the rest of the class. This would mean that they have now completed two parts of the data handling cycle.

Following this, the learners would need to represent the data in a pictograph. When creating a pictograph, you need to select a picture to use. As the data is on the different shapes found in the picture, it would be sensible to choose the same shapes to represent the number of shapes. The idea of using pictures, is so that it is easy to see at a glance which shape was used the most in the picture.
Finally, the learners could have an opportunity complete the last part of the data handling cycle by answering questions that encourage them to interpret the pictograph.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which shape has the most?</td>
<td></td>
</tr>
<tr>
<td>Which shape has the least?</td>
<td></td>
</tr>
<tr>
<td>Which shape has 2 more than the ○?</td>
<td></td>
</tr>
<tr>
<td>How many ○ and △ are there altogether?</td>
<td></td>
</tr>
</tbody>
</table>

Collecting, sorting and representing data – tallies and bar graphs

A bar graph is a graph made of vertical columns, each represents one category (e.g. type of fruit). Use graph or grid paper to draw your bar graph on, as this makes it much easier to keep your scale consistent without having to go to any effort measuring. As you can see, the bar graph looks similar in some ways to the pictograph discussed previously (pictographs show small icons that give a visual effect similar to columns or bars of different heights).

Here is an example of a bar graph to represent the data from a table.

<table>
<thead>
<tr>
<th>Occurrence of the numbers thrown with a die</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>31</td>
</tr>
</tbody>
</table>

ACTIVITY 2

Watch the video “Data Handling” (3:15 minutes) where a teacher collects data from her Grade 3 class by using tallies.

- Which parts of the Data Handling cycle are evident in the video?
- How does the teacher collect the data?
- How is the data organised?
- Is the data reported on and discussed in the video?
Commentary

Tally marks are a great way for learners to collect information. Many learners have a tendency to draw dots, and then become very confused when trying to count dots that are scattered all over the page. Tally marks teach learners to organise their data into a logical way, and it also helps them to count their data by skip counting in 5s. Remind learners that they need to draw 4 lines next to each other, and then the fifth line is a diagonal line through the other four to indicate a complete group of five.

In this video, we can see that three of the four parts of the data handling cycle are covered. The teacher collects data from the learners. The learners need to raise their hands and she counts the number of hands. The teacher then uses tally marks to organise the information. There is also an opportunity to discuss the tally marks, and to comment on the number of learners votes that they represent.

Interpreting data - pictographs and bar graphs

In the final part of the data handling cycle, learners are expected to analyse and interpret the data indicated on the graph. There is a difference between reading information off the graph, and actually interpreting the graph.

Reading information means that the information is clearly indicated on the graph, and that learners just need to look for it and read it off. Questions that require learners to read information from the graph would include:

- How many learners like oranges?
- On what day did the highest number of cars drive past the gate at the repair shop?
- Which is the least favourite fruit?

It is clear that these questions require an understanding of graphs, and that learners will need to know where to find the information. They will need to know that the x and y axes indicate
different information, and that they must look for the answers in the correct place. However, if they do understand the layout of graphs, then answering these questions becomes relatively simple.

Interpreting questions is a little more challenging, as the information needed to answer the question is evident in the graph but it is not as explicit as it would be for the questions discussed above.

Interpretation usually involves explaining or giving reasons for an answer, not just stating an answer. Referring to the graphs shown above the teacher could ask questions such as:

- Why do you think more cars drove past the repair shop on the weekend?
- Why do you think so few learners like bananas?

Another example of this would be when the teacher asks questions relating to a class graph on favourite pets. On the graph, it is shown that there are 7 dogs, 5 cats, 3 hamsters and 0 snakes. The teacher could ask “What can you tell me about the snakes?” A learner who reads the graph may answer that there are 0 snakes on the graph. A learner who interprets the graph may respond by saying “Snakes are the least favourite. No one chose them because they are scary”. In this response, the learner has recognised that there are zero snakes on the graph because no one voted for them, and the learner gave a possible reason for this.

ACTIVITY 3

Tallies and bar graphs

Watch the video “Tallies and bar graphs” (3:27 minutes) of a Grade 3 class using a bar graph to find out more about the favourite colours of the learners in the school.

- Which parts of the Data Handling cycle are evident in the video?
- How do learners interpret the data in the bar graph?
- How is the bar graph different and similar to the pictograph used in Lesson 9, Activity 1?

Commentary

In the video the teacher is working with a bar graph which represents the favourite colours of the learners in the school. She encourages the learners to interpret the graph by asking them questions about the represented data. She asks questions that get the learners to read the graph, where they can look at the axes of the graph and easily identify the number of learners who voted for a particular colour. The teacher also asks questions to encourage the learners to think a bit more about the data. When she asks which colour T-shirt they should order, the answer is not immediately evident. The graph identifies favourite colours, so the learners need to think a bit about what this means in terms of buying T-shirts. They should be able to come to realisation that if blue is the most popular colour, then it would be logical to order blue T-shirts as most learners like that colour.
In both Lesson 9 Activity 1 and Lesson 10 Activity 3, the learners were looking at data regarding the favourite colours of learners in the school. Although the actual data collected is different, the purpose of the graph in both activities is to represent the favourite colours of learners in the school. In the activity in Lesson 9, a pictograph is used. As we have already discussed, a pictograph uses pictures to represent data. When representing colours, the teacher did not need to choose a picture as such, but she rather just used a block of colour. This means that the pictograph and the bar graph look quite similar, although there are clear differences. The pictograph has individual blocks of colour as ‘pictures’, whereas the bar graph has long bars of colour.

Check your understanding: True or False?

1) The pictures used in a pictograph can be unrelated to the data being represented.
2) A bar graph can be made of different pictures.
3) Tally marks can create confusion when learners try to organise their data.
4) Interpreting data requires some understanding of the information recorded on the graph.

REFLECTION

• Reflect on your experience of teaching Data handling – in what ways does data handling provide a context for the learning of other mathematical concepts?
• What other skills and concepts (in addition to mathematical concepts) do you think could be developed through data collection activities?

Well done you have completed Lesson 10.