NUMERACY ACADEMY

PART 4:

SPACE AND SHAPE, MEASUREMENT AND DATA HANDLING

TEACHING CHILDREN MATHS

10 HOURS
Numeracy Academy

A team of writers from Bala Wande developed the Mathematics content of the Numeracy Academy drawing on the Bala Wande Thinking Maths modules, in consultation with Cally Khune of RED INK. The materials also draw on the Bala Wande Foundation Phase materials (Grades R to 3) were developed in consultation with a reference team of early Mathematics specialists.

www.fundawande.org
www.redink.org.za

Version 1.0: 2023

Anyone is free to share (copy and redistribute the material in any medium or format) or adapt (remix, transform and build on the material for any purpose), provided that you credit the work as follows:

Bala Wande, Numeracy Academy, CC BY 4.0.

You may not add terms or measures that legally restrict others from doing anything the licence permits.

For more information: https://creativecommons.org/licenses/by/4.0/
Acknowledgements

Layout and design:
Mandy Darling
Magenta Media

Citation:
Funda Wande: Cape Town.
The following materials were used in the development of this course:

Bala Wande Grade R Mathematics Programme Teacher and Teacher Assistant Training Facilitator Guides. (2021, 2022).

Bala Wande Grade R-3 Mathematics Programme Teacher Guides and Learner Activity Books. (2021, 2022, 2023).


Saide (2008). ACE Maths Unit 5: Building assessment into teaching and learning, from Teaching and Learning Mathematics in Diverse Classrooms. Saide: Johannesburg


Welcome and Orientation

Welcome to the Numeracy Academy – this course aims to get you to start Thinking Maths!
The materials are divided into 4 parts. Each part should take you 10 hours to complete.
It is important that you go through the lessons in sequence as each lesson builds on the content
from the previous one.
We encourage you to be an active reader while engaging with each lesson.

Each lesson has **video(s)** that you need to watch by clicking watch now. If you are reading the
print version of the booklet you can use the QR code to access the video.

Each lesson also has a **self assessment** that you should complete. This will give you a chance to
recap on what you learned in the lesson.

Each lesson ends by providing you an opportunity to **reflect** on what you have learned in the
lesson. Take time to do this activity - it will help you consolidate and action the learning in your
classroom.

**REFLECTION**

- Reflect on your own school experience as a child. Did you have opportunities to play as a means
  of learning?
- Think about your own practice. Does your teaching provide opportunities for children to play? Or
  are your lessons mainly teacher-led? Write a personal goal in relation to enhancing the teaching
  and learning in your classroom by including playful experiences.
In order to gain the most from this course, please ensure that you watch the videos in full and that you complete each self-assessment. The assessments during the course are self-checks and the answers are given at the end of each lesson. As part of the final assessment of the module there will be two tests.

- Test 1 is taken after the completion of Part 1 and 2.
- Test 2 after the completion of Part 3 and 4.

- Each test lasts 1 hour and is in multiple-choice format.
- An online link for each test will be provided on the scheduled date.
- You will receive your results after clicking the submit button at the end of each test.
- If you fail the test you will be provided a second chance to take the test and a new date will be scheduled for this.

We hope you enjoy the course and find it beneficial!
PART 4:

SPACE AND SHAPE, MEASUREMENT AND DATA HANDLING

LESSON 1: Space and shape – naming and recognition of 2-D shapes ................................................... 6
LESSON 2: Space and shape – working with 2-D shapes ........................................................................ 12
LESSON 3: Space and shape – 3-D shapes and transformations .............................................................. 17
LESSON 4: Space and shape – classifying and sorting 3-D shapes .......................................................... 23
LESSON 5: Space and shape – symmetry, position and direction .............................................................. 28
LESSON 6: Measurement concepts, units of measurement and problem solving .................................... 34
LESSON 7: How do we measure? ............................................................................................................. 39
LESSON 8: Conservation tests and the teaching of time ........................................................................... 44
LESSON 9: Data handling – collecting, sorting and representing data .................................................... 50
LESSON 10: Data handling – sorting, representing and interpreting data .............................................. 55

In Part 4 you will be introduced to the topics Space and shape, Measurement and Data handling. The focus on play-based learning, interactive teaching and using error analysis to inform your teaching will continue. In this module you will first learn about the teaching of Space and shape – one of the core topics of early grade maths learning that enables learners to orientate themselves in space, learn the language related to the topic and start to reason about spatial concepts. You will then learn about the teaching of the measurement topics – time, length, mass, capacity and volume. Finally you will learn about the teaching of Data handling, where a focus on the data handling cycle and how this teaching can be integrated with many of the other concepts taught in the early grades will be introduced.
In this lesson, we will discuss the topic Space and shape, outlining the intended outcomes for early grade learners. We will also learn about Van Hiele's Levels of Geometric Thought which is a theory on how learners learn about Space and shape. We will then look at 2-D shapes, considering what makes up shapes, and what makes one shape the same as or different from another. We will use this investigation to help us with the process of naming and recognising 2-D shapes.

There are many people who are unfamiliar or uncomfortable with this field of mathematics. They sometimes see it as inaccessible and impossible to master. If you are one of these people (to whatever degree) then hopefully the Space and shape lessons covered in Part 4 will help you to consolidate your understanding of the topic, and to develop an enthusiasm for teaching it.

What you will learn in this lesson
- The goals of the Space and shape topic
- Van Hiele's Levels of Geometric Thought
- Naming and recognition of 2-D shapes

The goals of the Space and shape topic

Geometry (or Space and shape as we now call it) focuses on the teaching and learning of lines, points, shapes and surfaces. As part of this topic, it is essential to understand the terminology related to the naming, recognition, sorting and classification of shapes (both 2-D and 3-D). There are four main outcomes for the topic of Space and shape, and these are covered across various levels of education.

Shapes and properties
- Look at properties of 2-D and 3-D shapes
- Investigate the relationships built on these properties

Transformation
- Translations
- Reflections
- Rotations (slides, flips and turns)
- Symmetry

Location
- Look at how object is located in space

Visualisation
- Recognise shapes in the environment
- Relationships between 2-D and 3-D objects
- Recognise objects from different perspectives
ACTIVITY 1

Look at the pictures below.

A  B  C  D
E  F  G  H
I  J  K

• Can you describe each picture?
• What characteristics do you notice?

Commentary

Whether you could name the shape or not and whatever characteristics of the shape you could give, if you look again you will quickly notice (if you have not already) that some of the shapes are flat and some of the shapes protrude into space. All these drawings are on a flat page - showing learners the actual objects will help them to understand the difference between shapes that are flat and those that are not.

This is the first major distinction that we are going to make in terms of geometric figures - some are called plane figures (they are flat and lie in a plane or flat surface; examples are B, C, E, F, H, I and J above). Others are space figures (they are not flat and protrude from the surface on which they are resting; examples are A, D, G and K above). Plane and space are separate from the dimension of the shape - they tell us whether the shape is flat or not.

Van Hiele’s Levels of Geometric Thought

This theory provides a major contribution to our understanding of how learners learn about geometric concepts. The model is divided into five levels, which are unrelated to our school grades. The levels are not age dependent, but they are sequential and describe a hierarchy of thinking processes. This is an important aspect to remember, as it means that a learner cannot progress to a new level of understanding without having moved through the previous levels first. It also means that you could have different learners in your class operating on different levels of understanding.

Within each level, the focus is separated into two parts called the objects of thought and the products of thought. The products of thought in each level is the same as the objects of thought in the next level, which indicates the progression of learning. Learners need many opportunities to be involved with activities at their current level of understanding. By doing this, learners consolidate their existing understanding whilst starting to explore the ideas of the next level.
As with other mathematical concepts, language is extremely important in the development of learners' geometric understanding. If the language used is at a higher level than the learner can cope with, then they will be unable to understand the concept being developed. It is possible to memorise facts about shapes without fully understanding the concepts or content. Learners need to be encouraged to talk about their learning so that they can clarify their knowledge as they share their ideas. The levels do not fit grade levels neatly, but could be categorised in the following way, to show the progression from one level to the next.

**Van Hiele's Levels of Geometric Thought**

<table>
<thead>
<tr>
<th>Level 0 – Visualisation (Gr R – 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognising and naming shapes</td>
</tr>
<tr>
<td>Working and playing with 2-D and 3-D shapes</td>
</tr>
<tr>
<td>See the shape as a whole: sorting, tracing, matching and drawing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 1 – Analysis (Grades 1 – 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties of shape</td>
</tr>
<tr>
<td>Talking about shapes</td>
</tr>
<tr>
<td>Verbalisation (social and academic language)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2 – Informal deduction (Grades 4 – 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classifying</td>
</tr>
<tr>
<td>Identifying common properties</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 3 – Deduction (High School and further education)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Level 4 – Rigor (High School and further education)</th>
</tr>
</thead>
</table>
ACTIVITY 2
Look at the shapes below.

- How are the shapes similar?
- How are the shapes different?

Commentary
In Level 0 of Van Hiele's levels of Geometric Thought, the objects of thought are shapes and what they look like. Learners would look at the shapes in the picture and be able to recognise and name them. Learners would focus on sorting and classifying the shapes based on their appearance. The products of thought are groups of shapes that seem alike. Learners need multiple opportunities to observe and feel shapes, taking them apart and building them back up again. Learners can discuss the properties informally, using their own language, without using all of the correct terminology.

It is important to teach at the learners level, however almost any activity can be adapted so that it covers two levels. You should ensure that learners have opportunities to use physical materials and to draw shapes at every level. Let learners sort the shapes according to their own criteria, looking for numbers of faces, edges and vertices, straight sides, round sides, and combinations of these.

Naming and recognition of 2-D shapes
In the early grades, language acquisition takes place at the same time as conceptual development. You always need to be aware of how language is being used in the classroom, and you should listen to and observe learners closely to work out what you think is best for them. There are many different ways to approach the teaching and learning of shapes, and variation is often the key.

Shapes that are 2-D (two-dimensional) are found in a plane. This means that they are flat and that they do not take up space. The CAPS makes a clear distinction between 2-D and 3-D shapes by referring to those that are 2-D as 'shapes' and 3-D as 'objects'. In fact, all objects in mathematics, can also be called shapes. The two dimensions refer to length and breadth or width. Early grade learners should explore the properties of the following 2-D shapes: circles, squares, triangles and rectangles. Learners should be encouraged to participate in activities that involve recognising, naming and describing 2-D shapes. They should also become accustomed to comparing and sorting shapes and using them to create other shapes.

Shapes are interrelated, and when learners become aware of this, they will start to see the shapes as less static and rigid, which is an important progression in their awareness of shapes. You could use paper-folding activities in your teaching on 2-D shapes. This would be a hands-on type of activity through which the learners could discover the characteristics of and inter-relations between shapes. Try this out and then record what skills and opportunities you think paper-folding exercises would offer to your learners. Early grade learners should be allowed to take time to establish that a square is always a rectangle and a rectangle is sometimes a square.
ACTIVITY 3

Naming 2-D shapes

Watch the video "Naming 2-D shapes" (3:26 minutes) where the teacher discusses a triangle with her Grade 2 learners.

• How are the learners involved during this activity?
• What language is being developed in the activity?
• What do you notice about the descriptions of the triangle?

Commentary

As we know, language is extremely important for learners understanding of mathematical concepts. Learners need to know the necessary terminology and be able to use this terminology to describe and define shapes.

You could investigate relationships between the following shapes in order to establish the truth of the following statements (note that this kind of knowledge would be useful for you as a teacher of mathematics and some learners may be also able to tap into this level of analysis but you would not teach it directly in the early grades):

• A square is always a rectangle.
• A square (and a rectangle) is always a trapezium.
• A rectangle is always a parallelogram.
• A parallelogram is sometimes a rhombus, when it has all four sides equal in length.
• A parallelogram is sometimes a kite, when it is a rhombus.
• A kite is sometimes a square, when both pairs of adjacent sides are equal in length and opposite angles are right angles.
• A rhombus is sometimes a square, when it has right angles.

In the topic Space and shape, a polygon is a simple, closed plane shape made only of line segments. Polygons can have many sides (line segment edges), and so they are named according to the number of sides that they have. Polygon means “many-angled”. The corners of a polygon are called vertices. The number of sides of a polygon corresponds with the number of angles of that polygon.

Mathematical definitions involve the use of mathematical terminology. Therefore, learners need to know the correct terminology and use it often. If the learners in your class are having difficulty with mathematical definitions – check and see if they understand each of the terms involved in the definition and you may find the root of their problem. Be very sure that you always use the correct mathematical language yourself, so that through listening to you, your learners will become acquainted with the terminology: you have to set the example! As mentioned in parts 1, 2 and 3 of this course, knowing the mathematical language and fluency of language (and procedures) is critical aspect of knowing and doing mathematics.
### Check your understanding: Multiple choice

1) **Which one of these is **not **one of the goals of the Space and shape topic:**
   - A) Look at shapes in the environment.
   - B) Fold paper to understand symmetry.
   - C) Look at the properties of 2-D and 3-D shapes.

2) **In Van Hiele’s Levels of Geometric Thought, there are:**
   - A) Six levels
   - B) Four levels
   - C) Five levels

3) **The level that most Gr R – 3 learners will operate on is:**
   - A) Level 0
   - B) Level 1
   - C) Level 2

4) **The dimensions used in 2-D shapes are:**
   - A) Height and depth
   - B) Width and depth
   - C) Width and length

### REFLECTION

- Reflect on your experience of learning about 2-D shapes at school.
- Think about how you teach 2-D shapes to your class.
- Write a personal goal with regard to helping the learners in your class to recognise and name 2-D shapes.

**Well done you have completed Lesson 1.**
In this lesson, we will discuss using 2-D shapes in geometric patterns. We will revisit our knowledge of patterns, which was covered in Parts 1, 2 and 3, and we will extend this understanding to include patterns involving shapes. We will also broaden our learning of patterns to include tessellation, and we will discover more about the use of tessellations in everyday life. Finally, we will investigate sorting and comparing 2-D shapes by looking at the features of shapes.

What you will learn in this lesson

- Geometric patterns
- Tessellation
- Sorting and comparing 2-D shapes

**Geometric Patterns**

As we have discussed in Parts 1, 2 and 3 already, patterns are an important part of Maths. Learners need to develop their ability to recognise and understand patterns so that they can use these to help them solve mathematical problems simply and efficiently. A pattern is a sequence of repeating objects, shapes or numbers. Patterns have rules which help us to determine which object, shape or number belongs in the pattern and which do not belong to the pattern.

In the pattern above, we can see that three horizontal lines follow three vertical lines each time. This makes it easy to determine that the next figure in the pattern will be three vertical lines. Patterns are all around us in our everyday lives, and it is helpful for learners to become confident in identifying these patterns. They can look for patterns in the paving, in the trees, on the floor, in the windows or in their clothes.

There are different types of patterns, and it is necessary for learners to recognise these patterns so that they can identify and apply the rules in order to complete the patterns. When completing patterns, learners will need to do some guesswork at first, but then they can use the rules to check if the pattern has been completely correctly.
The first type of pattern is the repeating pattern, where the rule keeps repeating over and over. The most important aspect to identify in a repeating pattern is the ‘unit’ (also called the core) and the number of elements in the unit because this is the part that repeats.

The second type of pattern is a growing pattern, where the figures increase as the pattern progresses.

The third type of pattern is a shrinking pattern. In this pattern, the figures decrease as the pattern progresses.

It is important to note how growing (and shrinking) patterns start and how they grow (or shrink) by a constant change or by a changing amount.

**ACTIVITY 1**

Watch the video “Geometric patterns” (3:14 minutes) where the teacher discusses patterns with her Grade 2 learners.

- What type of pattern is being used in the lesson?
- What could you do to vary the types of patterns being used in the lesson?
- Why do you think these types of patterns are important in Maths?

**Commentary**

In the video, the patterns demonstrated are repeating patterns. The learners constructed their own patterns, using colour and number of blocks to establish the rules for the patterns. It was a good idea to get the learners to construct their own patterns, as this gave them the opportunity to think about what they understood about patterns, and to create their own rules. By explaining their pattern to the teacher, they had to check that their rule actually worked for the whole pattern.
Perhaps the teacher could have extended the activity by encouraging learners to use a growing or a shrinking pattern, and to think about how these would change their rules. Working with patterns will help learners to develop their understanding of numerical patterns. Numerical patterns can be a sequence of numbers that involve calculations. For example in the pattern:

12 17 22 27 32

The rule is that you must add 5 to each number.

The identification of rules and patterns helps learners to simplify the solving of problems, because they become able to find solutions (or parts of solutions) mentally. They do not need to labour through solving each step, because they have an existing knowledge of number facts and patterns on which they can draw.

**Tessellation**

Tessellation is the art of covering an infinite surface without leaving any gaps between the shapes used to cover the surface. An example of this is the way we tile a floor or wall. If a single shape can be used to cover an infinite surface, then we say that that shape can tessellate. For example, squares can tessellate. If we cover a surface using a pattern that involves more than one shape, we call that a multiple shape tessellation. Tessellations can be made out of simple geometric shapes but also out of complex and creative shapes. Historically these patterns and designs go back as far as 4000BC. There is lots of evidence of them in Moslem and Islamic cultures in tapestries, tiles, rugs and quilts.

Give learners multiple opportunities to work with shapes and to make patterns. The properties of shapes become more apparent as learners fit them together, matching and rotating them to create patterns.

**ACTIVITY 2**

Think about tessellation and shapes that can tessellate.

- Will any triangle tessellate?
- Will any quadrilateral tessellate?
- Will any polygon tessellate?
- Experiment with tessellations of polygonal shapes to find out which of them tessellate. Cut out about six of any polygon that you wish to tessellate and see if it will tessellate. Paste down your tessellations on paper and keep them for future reference.

**Commentary**

As we have learnt, tessellation involves placing shapes together without any gaps to cover an infinite surface. It is interesting to note that only a limited number of shapes can form regular tessellations, without assistance from other geometric gap-fillers. Here are some examples of simple shape tessellations and some interesting tessellations drawn by young learners.
There are shapes that cannot tessellate by themselves. For example, circles and ovals do not have angles, and you can clearly see that it is impossible to put a sequence of these next to each other without leaving gaps. With shapes like these, you would need to add in other shapes in order to create a tessellation. Look around for some examples and share them with your learners to encourage them to experiment using shapes.

**Sorting and comparing 2-D shapes**

Learners need to be given many different opportunities to establish their understanding of the similarities and differences between 2-D shapes. It is essential that they verbalise their observations, and so use of the correct terminology should be encouraged. You can ask questions such as “What can you tell me about these shapes?”. An open question such as this allows for the learners to develop their thinking and reasoning skills, and to use verbalisation to clarify their own thinking. This process will broaden learners understanding of the Space and shape concepts. They may provide responses such as:

- “That one is smaller.”
- “Both have 4 corners.”
- “The bottom and the top match.”
- “They both have 4 sides.”

Look at the list of terms below and see whether you know all of them. As a teacher you always need to know more than your learners do.

- **Line segment** - A line segment is straight and has two fixed endpoints.
- **Angle** - An angle is formed where two lines meet at a shared point. Angles are measured in degrees.
- **Vertex / Vertices** - A vertex is a point where two lines meet.
- **Regular** - For mathematical shapes it does not mean “common” or “often seen”. If a shape is regular, it means that the sides of the shape are all equal in length and the angles of the shape are all equal in size.
- **Polygon** - A shape with at least 3 straight sides and angles.
- **Quadrilateral** - A shape with four straight sides.

**ACTIVITY 3**

Think about what you know about 2-D shapes.

- What is the value of playing with, building with and drawing 2-D shapes?
- Describe one activity where learners can get this experience in the early grades.

**Commentary**

The terminology of Space and shape is one area in which you need to be confident. Giving learners opportunities to play with shapes while they talk about them will give them the opportunity to become familiar with the shapes and learn the terminology. One such activity could be a game of “Shape Snap” – where images and names of shapes are on the cards and learners play in pairs.

You can also encourage learners to interact with shapes and to develop their verbalisation skills by playing games such as Shape Bingo or Guess My Shape. For Shape Bingo, learners would have a bingo board with shapes on it. They need to listen to the explanations of the shapes and match it to the shapes on their board. The first learners to cover a whole row of shapes shouts...
out “Bingo!” To win the game. In Guess My Shape, learners would have to describe a shape hidden inside a bag or packet using only their sense of touch. The other learners would need to guess which shape it was, and the first learner to guess correctly would be called up to describe the next shape. Learners can also use a pictograph to sort and compare shapes [see Lesson 10]. They can arrange the shapes into the circles according to colour, size, how many sides, how many vertices, or whether they have straight or curved sides.

**Check your understanding: True or false?**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Patterns have rules which help us to determine which object, shape or number belongs in the pattern and which do not belong to the pattern.</td>
<td>True</td>
</tr>
<tr>
<td>2. A jumping pattern is one of the different types of patterns that learners learn about.</td>
<td>False – A growing pattern is one of the different types of patterns that learners learn about.</td>
</tr>
<tr>
<td>3. A circle is a shape that can create a regular tessellation.</td>
<td>False – A circle needs other shapes with it in order to tessellate.</td>
</tr>
<tr>
<td>4. An angle is formed where two lines meet at a shared point.</td>
<td>True</td>
</tr>
</tbody>
</table>

**REFLECTION**

• How confident are you with regards to your own knowledge of 2-D shapes and their properties?

• What could you do to extend your own understanding and to promote effective learning opportunities in your class?

Well done you have completed Lesson 2.
In this lesson, we will move on to looking at 3-D shapes. We will once again look at the naming and recognition of these shapes, considering whether the 3-D objects slide or roll as we investigate the features of 3-D shapes. As part of our discussion, we will continue to emphasise the importance of language and we will look at ways to get learners to actively participate in constructing their own understanding. We will also look at transformations, where we explore how shapes turn, flip or slide.

What you will learn in this lesson

• Naming and recognition of 3-D shapes
• Balls and boxes
• Transformations

Naming and recognition of 3-D shapes

As we move on to 3-D shapes, it is important to remember that 3-D shapes (or three-dimensional objects) take up space. They are not flat. The CAPS refers to these types of shapes as ‘objects’, however they can also be called shapes. The 3 dimensions refer to length, width and height or depth. It is necessary to give learners time to work with 3-D shapes in a variety of ways so that they can experience these dimensions for themselves.

Early grade learners should explore the properties of the following 3-D objects: prisms, spheres, cylinders and cones. You need to provide multiple activities that involve recognising, naming and describing 3-D objects. It takes time and practice for learners to be confident in these skills, and so they need to use physical resources to construct their understanding. Learners also need to practise comparing and sorting 3-D objects and building with these 3-D objects.

A 3-D shape is not a plane figure (a 2-D shape) but rather a ‘space figure’, because it takes up space rather than lying flat in a plane. These shapes have height which makes them protrude up above the plane in which they lie. They can be solid, made of surfaces, hollow skeletons (also called frameworks), or just simply a collection of points which are not flat.
The closed space figures that are made entirely of flat surfaces (such as cardboard or paper) are called polyhedra, or sometimes they are called polyhedrons. Polyhedra are three-dimensional, and are made entirely of:

- **Faces** – the flat surfaces (which are all polygons);
- **Edges** – where the faces meet (which are all line-segments);
- **Vertices** – where the edges meet (which are all points).

3-D shapes can be cut open and laid out into a flat 2-D shape. These flat 2-D shapes are called nets, and they can be folded up to make the 3-D shape. We can make nets for all of the polyhedra. There are also nets for some other space shapes which are not polyhedral (such as cones). It is a good idea to allow learners to work with nets, as this helps them to see how the 3-D shapes are constructed, and it also helps them to see how 2-D and 3-D shapes are connected.

**ACTIVITY 1**

**3-D Objects**

Watch the video “3-D Objects” (2:48 minutes) where the teacher helps the Grade 2 learners to recognise and name 3-D shapes.

- How are the learners involved during this activity?
- What language is being developed in the activity?
- What do you notice about the descriptions of the 3-D shapes?

**Commentary**

In this video, the teacher is teaching the learners the correct terminology for the 3-D shapes. She is describing the shapes for them as she encourages them to find the shapes in the classroom. It would be helpful to allow the learners the chance to describe the shapes for themselves, as this would help them to better associate the new terminology with the shapes.

Read the descriptions below and think about how confident you are in describing these 3-D shapes correctly.

- A **cube** is a 3-D shape with 6 equal faces.
- A **cone** is a 3-D shape with a circular base.
- A **pyramid** is a 3-D shape with a flat base that could be triangular, rectangular, square.
- A **rectangular prism** is a 3-D rectangular-based prism.
- A **cylinder** has two flat round ends and a round tube. It can roll.
- A **triangular pyramid** is a 3-D triangular based pyramid also known as a tetrahedron.
As we discussed with 2-D shapes, you can use games such as Shape Bingo or Guess My Shape to develop learners verbalisation skills. The learners would play the games in the same way as described in Lesson 2 except they would now use 3-D shapes rather than 2-D shapes. Learners can also use a pictographs to sort and compare 3-D shapes in the same way as they did for 2-D shapes. They could compare and sort 3-D shapes based on size, colour, curved or straight surfaces, or whether they roll or slide. Learners enjoy opportunities to build using 3-D objects. This type of construction, where they build using concrete materials, allows learners to explore the properties of 3-D shapes in a different way.

### Balls and boxes

Using real life 3-D objects gives learners a chance to explore the properties of 3-D shapes through physical experiences. Give each group some boxes and some ball shapes and encourage them to build towers with the items. As learners participate in the activity, they will discover that some objects cannot be placed underneath other objects. For example, they cannot put a ball underneath a box, because the box will fall down. It is important that learners discuss their constructions, and that they verbalise their predications in terms of the stability of each construction.

Learners can experiment to see whether it is possible to make towers by using only balls and only boxes or if a mixture of the two kinds of shapes would also work. They can also vary the height of their constructions, investigating whether or not the height of a tower influences its stability.

### ACTIVITY 2

**Slide and roll**

Watch the video “Slide and roll” (3:23 minutes) where a Grade 1 class investigates a variety of 3-D objects to determine if they slide or roll.

- Describe an activity where learners can investigate whether 3-D objects slide or roll.
Commentary
A practical activity would be best for this type of investigation, so you could take the learners outside to where there is a flat, smooth surface. You could use the corridor, the school hall or any other flat area which is convenient for you. Collect a variety of different sized balls (or spherical objects), boxes (or prism objects) and cylinders, and take these out with you. It would be a good idea to get learners to work in groups so that they can compare their findings. Try to keep the groups to a manageable size, so that all learners can see and participate in the activity. You can then give each group a collection of different objects from the ones that you collected and took outside with you.

Questions are extremely important as they encourage verbalisation and the development of learners understanding. Some possible questions include:

“Which of the objects do you think you can roll?”

“What are these objects called?”

“Why do spheres/ball shapes roll? (Because they have curved/round surfaces.)

“Why do box shapes slide?”

“Why do cylinders roll and slide?”

You can use your questions to help learners realise that some objects will roll, some objects will slide, and some objects will both slide and roll.

Transformations
We are now going to look at the idea of transformations. Transformations occur when a shape changes its position, its size or its shape. Transformations that do not change the size or shape of an object are called rigid motions. We will focus on the three rigid motions of translations, reflections and rotations.

Translations are when an object or shape ‘slides’ into a new position. When a shape has been translated, every point in the shape is moved the same direction and distance. You can involve the learners in activities by getting them to pull or push shapes ALONG a line, BELOW or ABOVE a line in order to draw translations. The shapes in the grid below have been translated. The orientation of the shape stays the same.
Rotate means TURN like you turn a door handle when you open a door. We can turn ANY object around, in a variety of ways: in space, in the plane, about a point (inside the shape), or about a point (outside of the shape). We can also turn the shape through any number of degrees. The rectangle below has been rotated through a few different degrees. You can see this because you see the same rectangle but lying at different angles. The orientation of the shape changes.

A reflection is a type of rigid motion in which the object or shape is flipped to create the 'mirror image' of the original shape. This transformation acts like a mirror, where all the points on the original shape are exactly reflected on the opposite sides of a line. The line is called the line of reflection. The reflected image has the same size and shape as the pre-image, but it faces the opposite direction.

The first and last image below show examples of reflections. The heart has been rotated and the semicircle has been reflected and translated.

ACTIVITY 3

• Why do you need to know more than just the information required to teach learners a particular content area?
• What is the value of playing and building with, and drawing 3-D objects?
• Describe how physical activity with shapes will help learners to develop their understanding of transformations.

Commentary

As teachers of young learners, it is easy to give in to the temptation of just knowing the information necessary to teach the learners in your class. Many teachers do not feel confident in their own understanding of Maths, and so they just know enough to get through each lesson that they teach. Unfortunately, this strategy doesn't allow for the possibility of questions or misconceptions that arise which then can't be addressed by the teacher. It is essential that teachers know more than they need to teach the set curriculum. Teachers’ own understanding should go beyond what they are explaining to learners, as their extended knowledge will help them to develop the learners understanding and reasoning skills.

Luckily, by working with concrete apparatus and being physically involved with activities, it is possible to extend our own knowledge and understanding of Maths as well as that of the learners. For many adults, we were taught maths by just completing problems and writing in workbooks. We may not have had much opportunity to work with concrete apparatus ourselves, which could have resulted in a limited knowledge base and reduced confidence. As we plan and prepare activities for the learners, where they can play and build with shapes and objects, we provide opportunities for the consolidation of understanding by physically seeing and experiencing each of the concepts being addressed.
### Check your understanding: Multiple Choice

<table>
<thead>
<tr>
<th>Question</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Which of these is not one of the dimensions in a 3-D shape?</td>
<td>A) Height</td>
</tr>
<tr>
<td></td>
<td>B) Volume</td>
</tr>
<tr>
<td></td>
<td>C) Width</td>
</tr>
<tr>
<td>2) The word used to indicate where two surfaces meet is:</td>
<td>A) Face</td>
</tr>
<tr>
<td></td>
<td>B) Vertex</td>
</tr>
<tr>
<td></td>
<td>C) Edge</td>
</tr>
<tr>
<td>3) Building towers is a worthwhile learning experience because:</td>
<td>A) Learners learn more about the properties of 3-D objects.</td>
</tr>
<tr>
<td></td>
<td>B) Construction is good occupation.</td>
</tr>
<tr>
<td></td>
<td>C) The activity helps learners focus and concentrate.</td>
</tr>
<tr>
<td>4) A translation is a:</td>
<td>A) Turn</td>
</tr>
<tr>
<td></td>
<td>B) Flip</td>
</tr>
<tr>
<td></td>
<td>C) Slide</td>
</tr>
</tbody>
</table>

### REFLECTION

- Reflect on your experience of balls and boxes. Have you experimented with sliding and rolling?
- Think about when you could include this type of activity in your own teaching.

---

Well done you have completed Lesson 3.
In the lesson, we will discuss sorting and comparing activities, where learners can consolidate their understanding of the properties and features of 3-D shapes. We will then look at how to extend learners learn so that they can begin to move from Level 0 to Level 1. Most learners from Grade R – 3 will be working in Level 0, but it is important that you are prepared to challenge learners who are ready to move to Level 1. Finally, we will consider some ways to ensure that we are meeting the needs of all the learners in our classes.

What you will learn in this lesson

- Sorting and comparing 3-D shapes
- Moving towards van Hiele Level 1
- Teaching for all learners

**Sorting and comparing 3-D shapes**

When learning about 3-D shapes, learners need multiple opportunities to sort and compare concrete resources. They need to experience holding real objects so that they can feel the faces and edges. By doing this they will learn to make comparisons as they talk about the similarities and differences between the shapes. They can also spend time fitting shapes together to make bigger shapes (composing shapes) or breaking larger shapes down to make smaller shapes (decomposing shapes).

Some practical activities that you can use in your classroom include letting learners construct 3-D shapes by using salt dough to model the figures or using rolled up newspaper with masking tape. They could also use straws or coffee stirrers with twist ties or elastics to tie the ends together. It is important to make sure that you build up your activities from easy to medium difficulty, before moving on to more difficult tasks.

As learners develop their understanding, they can make a list of the properties of 3-D shapes that they have investigated. They can come up with their own definitions for shapes, and as they start to learn the correct terminology, they can begin to include this in their definitions. The learners can then use this knowledge and understanding as they participate in a variety of construction activities.
ACTIVITY 1

Watch the video “Faces of 3-D objects” (6:53 minutes) where the teacher works with a Grade 1 class, learning about the faces of 3-D objects.

- What do you notice about the involvement of the learners?
- How does this involvement help the learners to develop their understanding and to correct any misconceptions?

Commentary

In the video, the learners are all actively participating in the lesson. They all have concrete resources in front of them, that they are using these to discuss the faces of the shapes. This type of activity takes a bit of planning and preparation, as you will need to collect the boxes in advance. However, it is a worthwhile exercise, as the learners can physically see the properties and features of the shapes that they are learning about.

You will notice that one learner struggles to count the number of faces on his box, despite the fact that he is touching the sides as he counts. The teacher asks another learner to show him, which is a great idea. Learners often learn well from their peers and giving them the chance to talk and share their ideas is a valuable way of addressing misconceptions. When the learners are given the opportunity to draw around their boxes, they can easily see the shape of the face. This helps them to see the relationship between 3-D objects and 2-D shapes. This activity will also help them to describe their 3-D shapes, as they start to recognise that they can say that a 3-D shape (a cuboid) has 2 square faces and 4 rectangular faces.

Moving towards van Hiele Level 1

When learners have developed their confidence in their understanding of Level 0 concepts and activities, you can then begin to prepare them to move towards van Hiele Level 1. As we discussed in Lesson 1, Level 1 is the Visualisation level, where learners recognise and name shapes. As they move to Level 1, which is the Analysis level, they begin to explore the properties of shapes. In this level, learners need ample opportunities to draw, build, make, put together and take apart two-dimensional and three-dimensional shapes. They need to focus on the properties of the shapes, rather than simply identifying the shapes. Learners can be extended by challenging them to test their ideas about shapes by comparing a variety of shapes to see whether they all have the same features. Ask questions that get learners to think about whether the statements they make about the properties of a group of shapes apply to all the shapes in that group.

Remember that learners will not all be at the same level at the same time, so you need to prepare your activities so that you can meet the needs of all the learners in the class. You can structure your activities so that learners at all levels of understanding will have tasks that both provide opportunities for success as a certain amount of challenge. With careful observation, it is possible to assess at which levels the learners are working. Listen carefully to their comments and observe their reasoning, as these will give you a good indication of their level of understanding.
ACTIVITY 2

Watch the video "3-D objects" (4:12 minutes) where the Grade 3 learners ask questions to find out which 3-D shapes are in the teacher's bag.

What terminology can you hear being used in the activity?

- Do you think this is a good way for learners to learn about the features of 3-D shapes?
- Give reasons for your answer.
- How could you extend this activity?

Commentary

In the video, the teacher is encouraging the learners to use the correct terminology to describe the shapes (as we identified in Lesson 3). The learners are likely to remember this activity, and they will be able to use the language confidently in follow up lessons, as they are enjoying the game-like activity. This activity is more playful than a worksheet or written task would be, and so the learners are actively engaged in trying to work out the correct shape.

The teacher could extend this activity by getting the learners to compare 3-D objects. She could hold up a cylinder and a cone and ask, “What is the same about these objects?” Learners would need to note that both shapes have flat and curved surfaces. She could then ask, “What is different?” The learners would have to note that the cylinder has two flat surfaces, but the cone has one flat surface. The cone also has one pointy end, whereas both ends of the cylinder are flat. The teacher could then do the same with other 3-D objects, comparing them according to their characteristics.

- Cube and rectangular prism;
- Cone and the pyramid;
- Cone and sphere;
- Prism/cube and pyramid;
- Sphere and cube.

Teaching for all learners

In South African classrooms, we are likely to have learners with a variety of different needs. These needs may be learning needs, language needs, or even social, emotional or economic needs. In order for learners to learn effectively, we need to identify these needs and try to accommodate them as best as we can in the classroom. The first thing we need to establish is what the learners have as their existing, or prior, knowledge. The knowledge that learners bring into a lesson influences their ability to take in new information. We need to create an environment that focuses on learning with understanding. This type of learning is based on connecting and organising knowledge around the main conceptual ideas. This means that we can start with what
learners already know, and then build onto this knowledge, helping them to find patterns and relationships between ideas in order to simplify their learning.

It is essential that you take the context into consideration. Think about what is relevant for your learners, and perhaps make modifications to the content or curriculum so that you can better meet the needs of the learners. However, remember that this does not mean that you can have lower expectations of your learners. You need to maintain high expectations, and challenge learners to be resilient as they learn new information.

In a learning-centred mathematics classroom (spoken about in the mathematics framework for teaching mathematics in South Africa, DBE, 2018), you need to provide clear explanations of concepts and procedures in order to address learners’ errors and misconceptions. Learners should verbalise their mathematical ideas, and work towards making connections between topics. In order to do this, you should select and design tasks that emphasise key mathematical ideas and encourage learners to actively participate in each activity.

ACTIVITY 3
Think about the learners in your class.
• What different needs can you identify in your class?
• What are you currently doing to meet the different needs of learners in your class?
• What else could you do to meet these needs more effectively?

Commentary
When you establish a learning-centred classroom (DBE, 2018), there are four main areas on which teaching and learning activities should be built. These are conceptual understanding, procedural fluency, strategic competence and reasoning.

In order to develop conceptual understanding, teachers need to design lessons and activities that provide opportunities for learners to grasp mathematical concepts, operations, and relationships. In the context of our space and shape topic, this specifically refers to the development of learners’ ability to learn key concepts in relation to 3-D objects, 2-D shapes and their properties.

When learners are procedurally fluency, it means that they are able to carry out procedures flexibly, accurately, efficiently, and appropriately. When learning about shapes, the learners should develop procedural fluency in their ability to recognise, identify and name 3-D objects.

Strategic competence refers to the ability to identify and use appropriate strategies to solve mathematical problems. As learners participate in activities that involve building with 3-D objects and making 3-D objects using 2-D shapes, they will consolidate their ability to recognise the properties of 3-D objects.

Teachers need to provide many opportunities for learners to develop their mathematical reasoning skills. This is an essential skill that includes logical thought, and the ability to reflect, explain and justify. In our topic on Space and shape, learners will be able to justify and explain the relationships between objects using the properties of 3-D objects.
Check your understanding: True or false?

1) Decomposing shapes means that you break down larger shapes in order to make smaller shapes.

2) In Level 1, learners need to focus on identifying the shapes rather than on the properties of the shapes.

3) Your context means that you can lower your expectations to meet the needs of the learners in your class.

4) Developing reasoning skills is part of a learning-centred classroom.

REFLECTION

• Think about your learning environment. Do you think it is representative of a learning-centred classroom?

• Write a personal goal in relation to your engagement with the learners in terms of developing their participation in class.

Well done you have completed Lesson 4.
In this lesson, we will explore the concept of symmetry. This can be a tricky concept for learners, and we need to be careful to make sure that learners fully understand it before giving them tasks that are too challenging for them. We will then look at the idea of position, where learners learn to identify where a shape is in space. Language is particularly important here, as we then move on to direction, where learners will need to describe how to move to an object using the correct terminology.

What you will learn in this lesson

- Symmetry
- Position
- Direction

Symmetry

Symmetry is division of a 2-D or a 3-D shape into identical parts. When we say symmetry, we mean line symmetry. Line symmetry is also called reflection symmetry (because it has a lot to do with reflections) and bilateral symmetry (because of the ‘two-sided’ nature of symmetrical figures). We say line symmetry because of the line of symmetry in 2-D shapes - the line (or axis) about which the symmetry occurs. When two points are symmetrical to each other we say that the one is the reflection of the other. Symmetry can be horizontal, vertical or diagonal, and a shape needs to have only ONE axis of symmetry to be symmetrical, though it may have MORE THAN ONE axis of symmetry.

The topic of symmetry lends itself well to practical activities in the classroom. You could use any of the following:

1. Folding and making holes in paper with a compass/pen nib.
2. Folding and cutting paper shapes. Experiment with one and more folds and cutting on the different edges, too. Let the learners predict what the shape will look like before they open it up.
3. Point plotting on a coordinate grid or working out the unknown co-ordinates of a given symmetrical shape.

4. Using mirrors with real objects (e.g. pencils, sharpeners) and with drawings (familiar or unfamiliar).

5. Paint blobs on one side of a piece of paper and then squash two sides of the paper together along a fold. Learners love to see what interesting symmetrical images they can produce.

6. Colour blocks in a grid in a symmetrical pattern.

ACTIVITY 1

Symmetry (3)

Watch the video “Symmetry (3)” (4:57 minutes) where the Grade 3 learners learn about symmetry by colouring in blocks on a grid.

- What terminology do you hear in this activity?
- How are the learners involved in the activity?
- What do you notice about the way that the teacher addresses the one learner’s error?
Commentary

Hands-on activities are best as they involve the learners, and give them an opportunity to construct their own understanding. The teacher used the words horizontal and vertical, and demonstrated with her hands to help the learners remember what these words mean. The learners need multiple opportunities to:

- identify shapes that are either symmetrical or not symmetrical;
- insert line/s of symmetry in given drawings;
- complete drawings to make them symmetrical; and
- create symmetrical designs

In the video, the teacher helped the learners to recognise that some of the blocks had been coloured in incorrectly. She encouraged the learner to realise which blocks needed to be coloured in instead, rather than just giving the correct answer herself. If learners are struggling to identify the symmetrical picture, then it would be a good idea to use paper folding or a mirror to help them. These activities make it a bit more obvious what should be on the other side of a line of symmetry, and learners find it easier than just looking at a flat drawing.

There is symmetry all around us, in our homes, in nature and in art. Ndebele art is particularly interesting to study since it is full of geometrical shapes, symmetry and other transformations. Learners could be encouraged to draw their own Ndebele artworks as shown in the pictures below, linking their learning about maths to cultural knowledge.

Position

One of the goals of the Space and shape topic is for learners to understand the concept of location. They need to be able to describe the position of a shape in space. They can do this by describing the shape’s position in relation to that of another shape, or by considering the shape’s position on a grid.

Learners are comfortable describing position or location in their everyday, social language. They do this all the time in conversation. However, sometimes this social language is not clear or specific enough. Learners may be inclined to say “The square is over there” which doesn’t actually give us any real indication of the location of the square. Learners need to be given opportunities to develop their vocabulary of position. One way that they can do this is by creating maps of familiar objects, where they can indicate the position of an item by referring to its proximity to another item. It is worthwhile to allow learners to construct these maps in a concrete way. They can use real items or multifix blocks to do this. The concrete items help learners to better develop their understanding.

Learners find bird’s eye view maps extremely difficult, and they will develop misconceptions if this is introduced too early. Concrete maps form a solid foundation on which learners begin to understand that pictures and symbols can be used to represent the real objects. Once this foundation has been established, learners can move on to learning about movement on a grid. They will have objects scattered on a grid, and they will learn to use the correct terminology to describe their movement across the grid to a specified object. Once learners have grasped the idea of location, they can then investigate the movement of shapes.
Watch the video "Position and direction" (2:38 minutes) where a Grade 2 class uses the correct terminology to describe the position of objects.

- Make a list of the position-related vocabulary that you hear.
- What could you do to get the learners to use the vocabulary more themselves?

Commentary

In this video, the teacher helps the learners to see that they can describe the position of objects by using position-related words. It is important to use concrete objects, so that the learners can move the objects around themselves. By doing this, they get the opportunity to see the differences in the position of the object, and to construct their own understanding of the new terminology.

It is a good idea to get learners to use this vocabulary regularly, rather than in just isolated lessons. You could use the time in between lessons, or when you notice learners are getting a bit tired and need a short break, to call out some instructions using the terminology. You could say, "Stand on the left of your chair. Put your pencil under your table. Put your book on top of the pencil". A few short instructions like this could work well as you change from one lesson to next, giving the learners a ‘brain break’ as well as an opportunity to practice the new language.

Direction

We have discussed the idea of position in maths, which is the ability to describe where one object is in relation to another object. In the video you watched for Activity 2, you would have noticed the teacher getting the learners to follow directions in order to move around the classroom. In Maths, direction involves describing the way in which we need to move, for example forwards, backwards, left or right. We also use our knowledge of fractions when working with directions. Learners will need to be able to turn a quarter turn or a half turn.
Learners progress from describing the position of objects and shapes in patterns, to understanding how to explain movement in a straight line, as well as rotations. They will use the terminology “clockwise” (turning in the direction of the hands of a clock) and “anticlockwise” (turning in the opposite direction of the hands of a clock), as well as “left” and “right”.

Once again, it is important to get the learners to physically move around themselves. By having to follow the directions, and actually move their bodies according to the instructions, they will better understand both the terminology as well as the necessity for clear directions. Learners enjoy working in pairs, where one learner gives the directions and the other learner has to follow the instructions in order to reach a designated point. If you have access to an open space where learners could move around safely, you may even challenge the learners further by blindfolding the learner following instructions. By doing this, the learner giving instructions needs to be very specific and clear, as the blindfolded learner cannot look where they are going in order to help find the way.

**ACTIVITY 3**

Look at the coordinate grid below. The $\text{X}$ is positioned at (3,2) and the $\text{O}$ is at (1,3).

**Coordinate Grid**

- Ask a friend to play Noughts and Crosses with you on the grid.
- The first person to get three in a row is the winner.

**Commentary**

Using a game to teach learners about location is a great way to actively involve them in the construction of their understanding. In the example in Activity 3, the learners would learn how to identify coordinates by referring to the X axis (the horizontal line) first, and then the Y axis (the vertical line). The coordinates are written in brackets, separated by a comma. In the early grades you could use a grid to help learners to determine the position of items. Learners will use the terminology to describe how they would move across the grid to get to a specified point on the grid. For example, to get to the airport you would need to move forward two places from the starting block. You would then need to move one block to the right.
### Check your understanding: Multiple Choice

1) **Shapes can have:**
   - A) One line of symmetry
   - B) Three lines of symmetry
   - C) Multiple lines of symmetry

2) **Which activity would be a good way to teach symmetry?**
   - A) Paper folding
   - B) Using a mirror
   - C) Both A and B

3) **Learners can develop their understanding of position by:**
   - A) Creating concrete maps using physical resources.
   - B) Writing sentences with the new terminology.
   - C) Listening carefully to the teacher.

4) **When learners learn about direction, a key factor is:**
   - A) Practice
   - B) Vocabulary
   - C) Context

### REFLECTION

- Think about the way you learned about symmetry at school? Would you teach it in the same way or differently and why?
- How do you think that you can help learners to learn about symmetry effectively?

---

**Well done you have completed Lesson 5.**
The topic of measurement (or size) is one in which we teach our learners how to measure. We need to ensure that our learners understand the measuring process fully. To do so, the first need to understand the measurement concepts (of for example length, mass, capacity and volume) before we begin to teach them the skills of using measuring instruments. Simply put - you need to know what you are measuring before you can measure it. In this lesson you are introduced to ideas across the continuum of the teaching of measurement: starting with the foundational concepts of measurement, the use of units of measurement and problem solving in measurement contexts. These ideas relate to the teaching of measurement across all grades. You need to understand and be able to apply all of them yourselves in order to teach the topic well.

What you will learn in this lesson
- Measurement characteristics or attributes
- The relativity of size
- Standard units and non-standard units
- Word problems in measurement contexts - mass

Measurement attributes
Copley (2010) makes the point that adults often think of measurement in terms of formulas, rulers, and other measuring tools while young learners encounter measurement in many contexts every day as they explore and try to make sense of their world. Measurement involves quantifying the physical characteristics or attributes of physical objects that have size or amount. When we quantify, we assign a numeric value to something. For example, we can say that a belt is 90 cm long, or a cup holds 250 ml of water, or the mass of the learner is 34 kg. We cannot quantify things to which we cannot assign a numeric value. For example, if the belt is black, we cannot say how black it is by giving a number.

Learners are aware of physical objects and their characteristics before they develop a concept of number and measurement. We must ensure that they fully understand the concepts (of length, mass, volume or capacity for instance) of the things that we measure before we teach them how these are measured. This is because the way in which we assign numeric values to quantities is by comparing them to other quantities similar to themselves.

The relativity of size
Things which are absolute cannot be measured in degrees. They are, or they are not. They stand as they are. Size is not absolute. Size is relative and it is arrived at by comparison. We could say that something is long. What does this mean? How “long” is long? On the other hand, what is short? Perspectives differ, and different answers to these questions exist. That is what we mean by “size is relative” - it is given in relation to something else. It is this property of size that we use to quantify things. We compare them to “standard units” of themselves. Relativity of size is an idea we need to communicate to learners, even if in an intuitive way, without referring to relatives and absolutes!
We measure physical quantities not physical objects

What do we measure? We measure the size (amount) of an attribute of a physical object. Length, for instance, may be great or small. We measure the length of an edge, not the edge itself. We can say the length of the edge of a table is 56 cm. (We do not say the edge is 56 cm.) We may measure the mass of a ball. Then we would say the mass of the ball is equal to 3 kg. (We do not say the ball equals 3 kg.)

This shows the need for careful use of language in this topic of measurement, so that we avoid speaking unclearly or ambiguously. We must say exactly what we mean, and give clear instructions to our learners, so that they will know to which attributes we are referring. We must not allow any confusion between a thing itself (a table) and its attributes (the edge of a table).

**ACTIVITY 1**

- Think about the learners in your class. Which of their physical characteristics can be measured and which cannot?
- Write out a few clear instructions to learners, calling on them to measure some different physical quantities:
  - Relating to a learner
  - Relating to a desk

**Commentary**

The learners in your class have physical characteristics such as eye colour, hair, a smile, and personality; they are made of substance which is attracted by gravity; they have academic ability, artistic ability, sporting ability, shoe size; they have height, take up space, and so on. Some of these characteristics have size or amount – we call these physical quantities: mass, weight, length, volume – these are the attributes that we can measure.

- How tall are they? (length)
- How much substance are they made of? (mass)
- How much space do they take up? (volume)

When we ask learners to measure things, we need to use unambiguous, clear language. We would not say: measure that boy - we would specify which of the boy's attributes should be measured. For example we could ask them to measure the height or mass of the boy. If we want to ask for measurements of a desk, we do not say measure that desk we should ask them to measure the height or width or length of the desk. Each of those has a different measurement and learners need to know exactly what should be measured each time. Hands-on activities comparing physical quantities introduce learners to the idea of a unit of measurement, as seen in the pictures below.
Standard units and non-standard units

We choose suitable units to measure with. The units must possess the property of that which we are trying to measure. For example, to measure the length of the edge of a desk we could use a pencil because it has length, or to measure the length of the sides of a square we could use blocks. Then the block is 1 unit and the length of the side of the square is 5 blocks (units) as shown in the first picture below. It would be problematic if non-standard units were used to measure formally, since they are arbitrary and people could choose so many different non-standard units. As seen in the second picture below, if the blue blocks are used to measure the length of the car it is 3 blocks (units) long but if the purple blocks are used it is 6 blocks (units) long.

There are certain accepted standard units used for measuring the attributes. In the drawing above a ruler is lined up against a paperclip and a pencil, and we can see that the paperclip is 4 cm long and the pencil is 15 cm long. Centimetres are one of the standard units that we teach when we teach about measurement. Learners need to learn the names of the units, what they are used to measure and how to use the instruments of measurement. But before they reach the stage of learning about standard units, learners need to learn about the measurement concepts and the vocabulary related to these concepts.

**ACTIVITY 2**

Comparing lengths

Watch the video “Comparing lengths” (4:26 minutes) of a teacher in a Grade 1 class.

- Describe the activity the teacher uses in this lesson.
- In what way does the teacher encourage the use of length vocabulary in the lesson?
- Did this lesson involve estimation?

**Commentary**

In this lesson the teacher uses pieces of string as a concrete aid to give learners the opportunity to compare lengths. She gives learners pieces of string to work with and asks them to sort them into order of length. Then she puts them on the board, sorted from longest to shortest and calls on various learners to come to the board to point out pieces of string according to various
comparisons she gives. She uses the terminology long, longer longest and short, shorter, shortest all with reference to the string. In addition to learners participating in the general discussion with reference to the display on the board, you could also ask learners in pairs at their desks to compare lengths of string, talking to each other and circulating to see that they use the language correctly.

This activity did not involve estimation but it is very important when teaching about measurement to give opportunities for learners to estimate measurements – especially at the time when they are learning about the concepts themselves.

**Word problems in measurement contexts – mass**

Once learners have established the measurement concepts and know how to work with them they will be called on to do word problems that deal with measurement units in various contexts. This will apply their understanding of operations as well as their knowledge of the units of measurement. Word problems that enable learners to think about mathematics are another critical component of the teaching and learning of mathematics – in the context of measurement (or other contexts).

**ACTIVITY 3**

**Working with units of mass**

Watch the video “Working with units of mass” (5:30 minutes) in which a teacher sets word problems for her Grade 3 class.

- Describe the activity the teacher uses in this lesson.
- How does the teacher encourage learner talk in the lesson?
- How would you add to/change the lesson if you did it in your Grade 3 class?

**Commentary**

The teacher has set some word problems for the class which she asks them to do. She tells them about the questions and then gives them time to work out the solutions. She calls on learners to answer questions, probing their methods of solution. Assumed knowledge here is that of multiplication and division – the teacher could go into deeper explanations of the methods of solution and also set more problems. She could also ask learners to make up their own questions of a similar nature and try to solve those together.
Check your understanding: True or False?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Size is absolute.</td>
<td></td>
</tr>
<tr>
<td>2) Getting learners to measure their table is a good activity in which to involve learners.</td>
<td></td>
</tr>
<tr>
<td>3) A pencil is a good example of a non-standard unit of measurement.</td>
<td></td>
</tr>
<tr>
<td>4) Word problems in context allow learners the opportunity to practice the operations while consolidating their understanding of measurement.</td>
<td></td>
</tr>
</tbody>
</table>

REFLECTION

- Think about your own practice of introducing the measurement concepts. Do you take adequate care to use correct language when asking learners to measure things?
- When you introduce the concept of length to learners do you give them a lot of opportunities to work with concrete manipulatives? Describe some of the activities you do.

Well done you have completed Lesson 6.
The topic of measurement should be taught using apparatus and practical exercises wherever possible. In the early grades, the conceptual understanding of the measurement attributes is established. Throughout this course we have emphasised the importance of teaching skills based on an understanding of concepts. We must remember this in our teaching of measurement and facilitate it by giving a good conceptual grounding followed by sufficient practical exercises. In this lesson we discuss further skills and concepts related to the activity of measuring. Your understanding of these concepts will help enrich your teaching of measurement.

What you will learn in this lesson
- What is measuring?
- Pure numbers and denominate numbers
- Precision of measurement
- Direct and indirect measurement
- Capacity and volume – how are they related?

What is measuring?
Measuring is the process whereby we assign a number to a physical quantity by comparing it with a standard physical quantity. In the previous lesson we discussed non-standard and standard units. In our teaching of measurement, we use non-standard units to assist the formation of the measurement concept we are teaching before we introduce the commonly accepted standard units applicable to that which is being measured.

Pure numbers and denominate numbers
Pure numbers relate simply to the concept of number, of how much, without concerning themselves with of what. Cardinal number concept (the focus of Part 1 of this course) is about pure numbers. Denominate numbers are a special category of numbers which specify what is being counted. This is an important distinction for you to remember in the teaching of size, since all measurements are denominate numbers.

When we give measurements, we must state the unit of measurement.
For example, we measure mass in kilograms (kg). The scale on the right measures mass in kilograms.
Learners need to know how to read a scale like this – they should be able to show (by drawing in the arrow of the scale) that the bag of flour on the right has a mass of 5 kg.
Can a measurement be precise?

Another question we need to think about in the teaching of size relates to the precision of measurement. Because size is relative, a measurement will be as precise as the measuring instrument you are using allows you to be. If you use the same measuring instrument to measure two separate quantities, your measurements will have the same precision. The precision of measurements is also affected by the instruments we use to measure. It is important to choose an appropriate unit of measurement since this affects the precision of a measurement. For example, we would not use teaspoons to measure the capacity of a bucket – teaspoons are too small relative to the size of a bucket. But we could use jugs to measure the capacity of a bucket.

A measurement could be faulty if the instrument is faulty. Human error can also lead to faulty measurements: simple errors of carelessness or incorrect reading of the instrument. Exact measurements are not possible, but a level of accuracy can be chosen which is appropriate to a situation.

**ACTIVITY 1**

- Why are all measurements given as denominate numbers? What does this mean for the teaching of measurement?
- Why do we say that no measurement can be exact?

**Commentary**

Measurements are always given as a number of units, whether non-standard or standard units have been used. You must teach your students to record their measurements correctly, giving the unit of measurement each time. Denominate numbers (units) need to be properly used when it comes to computations involving these numbers, once the concepts and measuring skills have been taught.

Measurements cannot be exact since there is always room for some error in the reading of a measurement – but we do try to measure as accurately as possible and we should teach our learners to do so as well. They need to know how to use the measuring instruments correctly and to record their readings correctly, using the unit of measurement.

**Direct and indirect measurement**

Certain quantities can be measured directly. Think about the length of a pencil or lengths of the edges of a square. A ruler can be used. Think about the volume of a cube. Unit cubes can be used. Most measurement activities in the early grades involve direct measurement.

But some measurement cannot be made directly. For example – how could you measure the perimeter of a piece of paper which has been cut into an irregularly shaped region, or the volume of an irregularly shaped stone? They cannot be measured directly, but both of these measurements can be found by using a procedure we call indirect measurement. This requires a certain level of abstraction and a clear understanding of the concepts involved if it is to be grasped.

Let us look at the example of the perimeter of an irregular region. Remember that perimeter is the measurement of the outside boundary of a shape. To measure an irregular perimeter we can use a piece of string. We take the string and lay it carefully all along the border of the shape.
Then we take the string away from the shape, cut it off so that we have the piece that will give us the measurement of the perimeter, straighten it up, and measure how long the perimeter of the shape is using the string and a ruler.

This is indirect because we did not place the ruler along the border of the shape, because we could not. We used an indirect method (string) which successfully enabled us to find out the perimeter of the shape.

To measure the volume of a stone we can submerge it in water and find out how much water it displaces. The amount of water displaced will equal the volume of the submerged stone.

**ACTIVITY 2**

• When you get into a bath, what happens to the level of the water in the bath?
  o Why do you think this happens?
  o What does this show you?

**Commentary**

This is another example of indirect measurement. If you step into a bath, you displace the water in the bath. The water level rises. This shows that your body has volume! It takes up space.

**Volume and capacity**

Volume is the measure of the amount of space an object takes up. Capacity is the measure of the amount of space inside, or the ability of an item to hold something. Everything that takes up space has a volume. Containers that are hollow and can hold things (such as liquids) are said to have capacity. When a container is full, we say it is filled to capacity. That means it is holding as much as it is able to hold. Volume and capacity are related. The liquid that you pour into a container has volume - the volume of water that it takes to fill a container is equal to the capacity of the container. Because of this relationship, volume and capacity can both be measured in millilitres, but volume is also measured in cubic units.

Solid objects also have volume - for example the cube illustrated above has a volume of 25 cubes. Your body has volume - as discussed in Activity 2 above - if you step into a bath your body displaced an amount of water which is equal to the volume of your body that has gone under the water.
ACTIVITY 3

Comparing volume and capacity

Watch the video "Comparing volume and capacity" (4:27 minutes) of a teacher in a Grade 1 class teaching about capacity.

- Describe the activity the teacher uses in this lesson.
- What other questions might you ask if you used this activity in your class?

Commentary

In this activity the teacher asks her class to sort a set of four containers according to capacity. The learner that comes to the front sorts them from the tallest to the shortest container and evidently believes that the tallest one will hold the most and the shortest one will hold the least. The teacher has filled the first container (a 1 litre coke bottle) with water and proceeds to ask the learner to pour the water from one container to the next. They see that these containers all have the same capacity - 1 litre. This is a nice activity to show learners not to be deceived by the height of a shape when thinking about the capacity of the shape. You could ask questions that call on the learners to think more about heights of the containers and their volumes. Remember to ask questions in such a way that learners are made to use the measurement vocabulary themselves and not just answer simple yes/no questions.

When you teach capacity you can easily collect empty containers and bring them to class to use in demonstrations, or even better, bring enough for all learners in the class to work in groups doing hands-on activities. In the activity suggested by the series of pictures below, the teacher works with her class to compare the capacity of some different containers using a glass. See how she made a funnel using a cut off bottle and involves the learners in the demonstration.
### Check your understanding: Multiple Choice

1) **Measuring is the process whereby we:**
   - A) assign a number to a physical quantity by comparing it with a standard physical quantity with the same attribute.
   - B) assign a number to a physical quantity by comparing it to a different physical quantity.
   - C) Estimate the size of a physical quantity by comparing it to another physical quantity.

2) **A measurement is:**
   - A) Perfectly accurate.
   - B) As precise as the measuring instrument allows.
   - C) Different depending on the time and place of the measurement.

3) **An indirect measurement is:**
   - A) A measurement completed by someone else.
   - B) A measurement recorded in a book.
   - C) A measurement completed by using a piece of string and a ruler.

4) **Capacity is:**
   - A) the measure of the amount of space an object takes up.
   - B) the measure of the amount of space inside an object.
   - C) The measure of the distance an object covers.

### REFLECTION

- Reflect on your understanding of the relationship between volume and capacity. What do you think is the best way to make this clear to young learners.
- Write a personal goal in relation to your teaching of measurement – in what way will you deepen your own understanding of measurement concepts so that you will be able to teach them with greater insight.

---

**Well done you have completed Lesson 7.**
Conservation tests give us a way to establish whether or not a learner has understood a certain measurement concept. In the next section we look at some introductory exercises that can be used in the establishment of the size concepts.

**What you will learn in this lesson**

- Conservation tests for the teaching of measurement - length and area
- Conservation tests for the teaching of measurement - mass
- Measurement of time

**Piaget’s conservation tests**

The ideas of the developmental psychologist Piaget about conservation of number are well known. In this lesson we use his ideas again, to check our learners’ readiness to proceed with the measurement of things such as length, mass, area, volume and capacity. As with number concept, we need to check that learners have achieved conservation of these concepts before we can teach about their measurement. Conservation of the concept means that they have a clear understanding of the constancy or unchanging nature of length, mass, area, volume and capacity. In other words, they understand the meaning of these concepts.

In this lesson we will look at conservation tests for each of length, mass area, volume and capacity. The tests follow a similar pattern but vary according to the measurement concept involved. For example, before we look at each test separately, we say that a learner has achieved conservation of length once they are aware that the length of a piece of string remains the same, no matter if we lay it straight, curve it, roll it up or even cut it up. So the conservation tests are all designed to check whether the learners know that equal amounts remain equal even when their appearances have been distorted.

Learners develop at different paces, and we cannot assume that they will all achieve conservation of the measurement concepts at the same time. It does not take very long to test for the conservation of these concepts, so we should always just take that little extra step to check for conservation before we proceed to teach how to about measurement and the use of units of measurement.

Piaget went further to say that if a learner is able to explain that the distorted amounts could be restored to their original appearance, then the learner has achieved the concept of reversibility, another affirmation of the understanding of the concept.

**ACTIVITY 1**

- Why are the conservation tests useful in the teaching of measurement?
- Give an example of what you think is meant by reversibility in relation to the concept of length.

**Commentary**

The conservation tests are useful because they help a teacher find out whether or not a learner has understood a concept. They are an effective way to test for understanding since the answers
learners give are the evidence of their understanding or lack of understanding. If a learner has achieved reversibility in relation to the concept of length it means that they know that a piece of string (for example) has been cut into pieces, the total length of the individual pieces will be same as the total length of the original piece of string.

**Conservation of length**

*Length is the measurement of the size (how long?) of an edge (straight or curved).* In the early grades, learners develop their understanding of the concept of length by comparing objects directly (for example by lining up two objects side by side to see which is longer). To check for conservation of length, show the learner two pieces of string that are the same length. Let her satisfy herself that they are the same length.

Now take one of the pieces of string and twist it around into a coil. Ask the learner if the two pieces of string are same length, or if their lengths are different (second display). You could then further distort the one piece of string by cutting it up into a few pieces (third display). Then ask again if the two displays contain the same length of string.

In the initial display, both pieces of string are the same length. In the second display, one piece of string is twisted into a coil, making it appear shorter. In the third display, the same piece of string is cut into pieces, making it appear even shorter. If the learner answers that the pieces of string are the same length, she has achieved conservation of length. If she answers no at any stage, then she is not sure that the length of the string remains the same even if its appearance is changed, and she has not achieved conservation of length.

If she can explain why they are still the same lengths in terms of restoring them to their original shapes, she has achieved reversibility of the concept of length.

**Conservation of area**

*Area is the amount of surface covered by a shape.* Learners develop their understanding of the concept of area by using a variety of materials to cover the surface of shapes. To test for conservation of area, show the learner two postcards which are exactly the same. They have the same area. Let her satisfy herself that they have the same area.

In the initial display, both postcards are the same size. In the second display, one postcard is cut into two parts, making it appear smaller. In the third display, the same postcard is cut into even smaller pieces, making it appear even smaller. If the learner answers that the two areas covered are still the same, or if they cover different areas. You could then further distort the one postcard by cutting it up into a few pieces (third display). Then ask again if the two displays still cover the same area.

Now take one of the postcards and cut it into two parts (second display). Ask the learner if the two areas covered are still the same, or if they cover different areas. You could then further distort the one postcard by cutting it up into a few pieces (third display). Then ask again if the two displays still cover the same area.
If he can explain why they still have the same area in terms of restoring them to their original shapes, she has achieved reversibility of the concept of area.

**Conservation of mass**

**Mass is the amount of matter of which an object is made**, although people often speak about the weight of an object, rather than its mass. Scientifcally, weight is a measure of the pull or force of gravity on an object. Young learners do not need to know this distinction but it is better for teachers to model the correct terminology and speak about mass (which is consistent with the CAPS curriculum). Learners in the early grades develop their understanding of the concept of mass through comparison activities. They might use their hands as ‘balance scales’, holding an object in each hand to determine which object is heavier.

To test for conservation of mass, show the learner two balls of clay which have the same mass. Let her satisfy herself that they have the same mass.

Now take one of the balls of clay and roll it into a thin sausage (second display). Ask the learner if the two pieces of clay have the same mass, or if their masses are different. You could then further distort the one lump of clay by cutting it up into a few pieces (third display). Then ask again if the two displays contain the same mass of clay.
If the learner answers that the lumps of clay have the same mass, she has achieved conservation of mass. If she answers no at any stage, then she is not sure that the mass of the clay remains the same even if its appearance is changed, and she has not achieved conservation of mass.

If she can explain why they still have the same mass in terms of restoring them to their original shapes, she has achieved reversibility of the concept of mass.

**Conservation of volume**

*Volume is the amount of space taken up by an object.* In testing for conservation of volume you could use the same balls of clay that you used in the tests for conservation of mass. Show the learner two balls of clay which have the same mass, and which therefore have the same volume. Let her satisfy herself that they have the same volume.

**ACTIVITY 2**

- What steps would you now go through to test the learner for conservation of volume? Explain in writing and illustrate your demonstrations for the initial, second and third displays.
- How would you know whether or not the learner has achieved conservation of volume?
- How would you check whether or not the learner has achieved reversibility of the concept of volume?
- What other apparatus would be useful in tests for conservation of volume?

**Commentary**

You would go through the same steps and in the same manner as the conservation tests for length, area and mass were carried out – but in with a focus on the volume of an object. Similarly, if the learner can answer that the volume does not change even if it has been distorted, he has achieved conservation of volume. If he can explain why they still have the same volume in terms of restoring them to their original shapes, she has achieved reversibility of the concept of volume.

In lesson 7 you watched a video of a lesson where learners established the fact that a group of containers (of different shapes) have the same capacity. This activity draws on learners understanding of the conservation of volume (that an amount of water, if poured from one container to another, even if it looks different, remains the same). This hands-on activity thus helps learners establish the capacity of the containers using volume. Learners might believe that the containers hold different amounts (volume) of water. They test this using water (a fixed volume), poured from one container to the next to discover/prove that the containers all hold the same amount – in other words, have the same capacity.
Measurement of time

We speak about time so often in our daily lives. Time is something that we measure but we cannot see it or touch it! Yet we are aware of the passing of time and we do measure it. Time is the duration of an event from its beginning to its end. Regular teaching about time is needed because time is abstract – we cannot touch it or feel it, but we are aware of its passing. Learners become aware of time early in their lives because of the regular cycles of night and day, the seasons and the years. Teachers should help learners estimate, measure, and describe the way time passes. Learners need to learn both how to tell the time (using clocks) and also how to calculate how much time has passed. Introductory activities on the teaching of time link to sequencing of events which help learners to think about the passing of time. Then learners need to learn about the units of time, of which there are many – since time is measured years, months, weeks days, hours minutes and seconds – and more!

ACTIVITY 3

Hours and half hours

Watch the video "Hours and half hours" (3:10 minutes) of a teacher teaching a Grade 2 class about time and telling the time. The teacher recaps learners’ experiences of time and the importance of being able to tell the time.

• What is the value of recapping learners’ experience of telling the time?
• What sort of clock does the teacher use in her lesson? What is your experience of learners working with analogue clocks?
• This lesson is about hours and half hours – to what other maths concept could this be linked?

Commentary

Awareness of time and the integral role it plays in our daily lives will motivate learners to develop their skills of telling the time and working with units of time. It will also help them to become more responsible. This teacher used an analogue clock with hands that moved correctly together – the value of this is that it is realistic and can help learners see how the hands move together. Many learners do not know how to tell the time using an analogue clock because they are more used to digital clock faces. Analogue clocks are still found and in maths teaching fraction concept (of halves and quarters for example) can be linked to the passing of time on an analogue clock.

The hour hand goes around the clock two times in one day.
12 hours and 12 hours is 24 hours.
The minute hand goes around the clock every hour.
There are 60 minutes in an hour.
30 is half of 60. When the minute hand points to the 6, we say half past.
Check your understanding: True or False?

1) Conservation refers to a clear understanding of the unchanging nature of length, mass, area, volume and capacity.

2) Area tells us how long the edge of shape is.

3) Weight is a measure of how heavy an object is.

4) Time is only measured in hours and minutes.

REFLECTION

- Reflect on the value of the conservation tests and how you could use them when you teach measurement concepts.
- What apparatus would you use to test for conservation of length, mass, capacity and volume?

Well done you have completed Lesson 8.

Answers:
1. True
2. False – Area is the amount of surface covered by a shape.
3. False – Weight is a measure of the pull or force of gravity on an object.
4. False – There are many units of time: years, months, weeks, days, hours, minutes and seconds.
Data handling is the topic in the curriculum that touches on statistical concepts in the early grades. In this topic we start to develop, represent and interpret statistical information. If, for example, someone says to you, 55% of the learners at their school are boys, while only 5% of the teaching staff are males, they have given you some statistical information. This might make you think about the ratio of boys to male staff. Statistics help us to summarise information and make comparisons. Statistical results are often shown as graphs as these visual representations show off the statistics more effectively than words or tables might do it. The graphs are meant to make the interpretation and analysis of the information represented easier.

In the first of the two lessons on data handling in this course, we will outline the data handling cycle and some of the terminology which are used in data handling. We will then think about why we should teach Data Handling in the early grades.

What you will learn in this lesson

• The data handling cycle
• Data handling vocabulary
• Why should we teach Data Handling in the early grades?

The data handling cycle

The Data Handling Cycle

Collect and organise data
• Collect and sort everyday physical objects.
• Collect data about the class or school to answer questions posed by the teacher.
• Organise data supplied by teacher or workbook/textbook
• Organise data in
  - lists
  - tally marks
  - tables

Represent data
Draw a picture of collected objects.
• Represent data in
  - pictograph (limited to pictographs with one-to-one correspondence)
  - bar graphs

Analyse and interpret data
Answer questions about data presented in
• pictographs (limited to pictographs with one-to-one correspondence)
• bar graphs

Discuss and report on sorted collection of objects
• Give reasons for how collection was sorted;
• Answer questions about
  - how the sorting was done (process)
  - what the sorted collection looks like (product)
• Describe the collection and/drawing
• Explain how the collection was sorted
ACTIVITY 1

Watch the video “Representing and interpreting data” (3:55 minutes) of a teacher working with a pictograph in a Grade 1 class.

- Describe the activity the teacher uses in this lesson.
- Which parts of the Data handling cycle are covered in this lesson?
- If the lesson were extended could other parts of the cycle be included? Which of them and how?

Commentary

In the video, the teacher used a pictograph to discuss the favourite colours of the learners at the school. The colours are represented by blocks of colour, and the teacher explains that one block corresponds to one learner’s vote. We can see that in this activity, data is represented in a pictograph with one-to-one correspondence. This means that the activity covers the ‘Represent data’ part of the Data Handling cycle. In addition to this, the learners answer questions about data presented in the pictograph. This means that the activity also covers the ‘Analyse and interpret’ part of the cycle.

In order to cover the other two parts of the data handling cycle, we would have needed to begin the lesson by collecting the data ourselves. The teacher could have asked questions, and the learners could have recorded the information about their friends’ favourite colours. Once the information had been collected, then the learners would have needed to organise it in a logical way so that it was easy to read. This would have covered the ‘Collect and organise data’ part of the cycle.

We could then have moved on to the ‘Discuss and report on sorted collection of objects’ part of the cycle. This would involve getting learners to provide reasons for the way in which they organised the data. Learners could describe the data collection and explain how they have organised the information. This would be similar to the way in which the teacher explains the pictograph to the learners in the video, but there would need to be greater involvement from the learners.
Data handling vocabulary

There is a lot of vocabulary related to data handling. You need to be sure that you know the meaning of the terms described below so that you can use them yourself and apply them when you teach data handling lessons. Examples are given for each term, followed by an activity for you to work through to apply your understanding of each of the terms.

**Data** is information that is collected relating to a given topic. For example, you might want to find out information about the birthdays of the learners in your class. When you collect the information about birthdays, you are **collecting data**.

**Raw data** is data which has been collected but not yet sorted out in any way. For example, when you collect the information about learners' birthdays you could use a class list to do so. You could write the date of each learner next to their name on the class list. When you have finished collecting this data, it will not be easy to tell, from just looking at the list, if more birthdays fall in January than in July, since you have not yet counted up the number of birthdays which fall in each month. Raw data needs to be sorted.

A very common method used to sort data is to **tally** the data. You could use tallying to sort the data in the raw data example given above. To do so, you would write a list of the days (Monday to Sunday) on a page, and then go through the class list, making a mark next to the correct day for each name on the list. The tallies could then be counted up.

The totals that you get when you add your tallies give you the **frequencies** for the particular data collected. You could check that the total number of learners in the class is the same as the total you get if you add up all of the frequencies, to be sure that your tallying has been correct.

When we sort data into categories, the categories usually give us a way to **group** the data, which helps us to make sense of the data. For example, we could sort the birthday information we have collected. We could probably not sort this data according to date, as there would be too many different dates! There is very little chance that we would have many learners in one class with the exact same dates of birth, and so there would be almost as many categories as learners in the class. We could more effectively sort the dates we have collected according to days of the week (this would give us seven groups) or according to month (this would give us 12 groups). This would **group** the data in a more sensible manner and lead to meaningful comparisons.

**ACTIVITY 2**

A teacher gives her class an activity to find out about the colours of the cars that pass the school.

- Do you think this is an activity that can be completed easily?
- What are the benefits of this activity?
- What are the potential problems with this activity?

**Commentary**

When deciding on an activity which requires learners to collect data, you need to think about your context. For example, if you wanted to get the learners to find out about the colours of cars that pass the school, you would need to think about how many cars actually pass your school during the school day. If your school is on a quiet road where perhaps there are 4 cars that pass all day, then this activity would not be worthwhile. Equally, if the bulk of the traffic passing your school drove by at 7am, then this activity would not be very successful as you would probably not be able to get the learners to collect the data at that time of the morning. However, if your school is in a prime position to watch the cars passing (for a relatively short period of time) then this could be a valuable activity. Learners could collect the information themselves by observing the cars driving past. They could record the colours of the cars using tally marks, and then organise
this information into a pictograph. Learners could also have an opportunity to interpret the information on their graph, and to communicate their findings to the rest of the class. This means that an activity such as this could allow learners to progress through all four parts of the Data Handling cycle.

**Why should we teach Data Handling in the early grades?**

Data handling is an important topic to cover in the early grades because it develops a variety of skills that can be applied to other areas of Mathematics and learning in general. While it can be a tricky topic for learners to grasp, it is still worthwhile because learners develop the ability to formulate and solve problems. As they collect, organise, describe and interpret data, learners learn to identify problems, and to find ways to resolve the problems independently of the teacher. This increased independence is a necessary step in their growth and development, and they will get further opportunities to develop in this way as they produce their own graphs. As learners learn to complete all the parts of the data handling cycle, they learn how to collect, organise and interpret information that they have sourced themselves. This teaches them to produce their own knowledge, rather than just waiting to consume knowledge provided by the teacher.

Another valuable aspect of the data handling topic is that learners learn that graphs are a means of communication. This ability to recognise that we do not just communicate through written and spoken words is an essential part of Maths. Learners need to understand that pictures, numbers and graphical representations provide an efficient method of communicating, and that they should be able to make sense of this information quickly and easily. By developing their confidence in interpreting information presented in different forms, learners will develop the ability to seek out information and answers for themselves, rather than just relying on information given to them by someone else.

**ACTIVITY 3**

- Do you think it is important to develop early grade learners’ independence?
- Give reasons for your answer.
- What could you do to develop learners’ independence when learning about graphs?

**Commentary**

Young learners can still be taught to be independent. It is imperative that we don’t under-estimate learners’ ability to think for themselves simply based on their age. Learners are capable of rising to the challenges set for them, and so we need to encourage them to think for themselves from a young age.

In the classroom, you can present the learners with appropriate problems that require data collection. Initially, it would be useful to give them guidelines for the data collection. You might point out to them some of the following things (depending on the problem which has been set):

- Data should be found using the correct sample. This means that the learners need collect information from the appropriate people.
- We cannot estimate, guess, or make up data. Actual data must be found.

If special instruments are needed in the recording of the data, ensure that the learners know how to use these instruments and check that the instruments are in good working order. This will help them work more independently. The learners might not know all of the graphical forms of representation, so you need to decide whether you will tell them which type of graph to use or if you will let them choose for themselves. Letting them choose for themselves is a great way to develop their independence, however you need to be sure that this meets your requirements in terms of the learning content.
## Check your understanding: Multiple Choice

1) Which of the following is not part of the Data Handling cycle?  
   A) Collecting data  
   B) Drawing data  
   C) Analysing data  

2) Raw data is:  
   A) Irrelevant data  
   B) Incomplete data  
   C) Unorganised data  

3) Frequency is:  
   A) The total that you get when you add your tallies.  
   B) How often you collect data.  
   C) The number of times you discuss your data.  

4) Graphs are a way of:  
   A) Communicating information efficiently.  
   B) Hiding information.  
   C) Storing information.  

## REFLECTION

- Why does the topic of Data handling lend itself to group work and projects?  
- Write a personal goal in relation to making your teaching of Data handling more relevant to the personal contexts and lives of your learners.

---

**Well done you have completed Lesson 9.**
In the second of the two lessons on data handling in this course, we will think about ways of collecting, sorting and representing information, this time in the context of tallies, pictographs and bar graphs. After this, the highest level of activity related to data handling in the early grades, interpretation of data, is considered. In this lesson we also discuss the ways in which Data Handling activities can be integrated into other topics in the curriculum, since Data Handling provides a context for the learning of other mathematical content.

What you will learn in this lesson

- Collecting, sorting and representing data - tallies and pictographs
- Collecting, sorting and representing data - tallies and bar graphs
- Interpreting data - pictographs and bar graphs

Collecting, sorting and representing data - tallies and pictographs

A pictograph is made up of little icons (pictures) which represent certain numbers of things as is indicated in the key, which must accompany the pictograph. The picture selected usually relates in some way to the data being represented. A pictograph looks and functions a bit like a bar graph, as you will see in the next section.

For a pictograph to show the number of cars passing through various tollgates on 24 September in a certain year, learners would be given a frequency table as shown below:

<table>
<thead>
<tr>
<th>TOLL GATE</th>
<th>NUMBER OF CARS PASSED THROUGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mooiriver</td>
<td>40 500</td>
</tr>
<tr>
<td>Grassmere</td>
<td>52 000</td>
</tr>
<tr>
<td>Kranskop</td>
<td>43 000</td>
</tr>
<tr>
<td>Middelburg</td>
<td>32 000</td>
</tr>
<tr>
<td>Kroonvaal</td>
<td>40 500</td>
</tr>
</tbody>
</table>

They could then use the data to create a pictograph to represent the number of cars passing through the tollgates:
ACTIVITY 1
Look at the picture below.

- Could this picture be used in a data handling activity?
- Which parts of the data handling cycle could be covered?

Commentary
This picture could easily be used in a data handling activity. It would work well because you could use it to cover all four of the parts of the data handling cycle.

To start off, the learners could collect the data from the picture by counting the number of circles, triangles, squares and rectangles in the picture. The learners would have to find a way to manage their data collection because they could get confused if they just count by pointing at the pictures. They may need to cross off shapes as they count them, or perhaps use tally marks to keep track of the number of shapes.

Once they have organised their data, learners can discuss and report on this data with the rest of the class. This would mean that they have now completed two parts of the data handling cycle.

Following this, the learners would need to represent the data in a pictograph. When creating a pictograph, you need to select a picture to use. As the data is on the different shapes found in the picture, it would be sensible to choose the same shapes to represent the number of shapes. The idea of using pictures, is so that it is easy to see at a glance which shape was used the most in the picture.
Finally, the learners could have an opportunity complete the last part of the data handling cycle by answering questions that encourage them to interpret the pictograph.

<table>
<thead>
<tr>
<th>Which shape has the most?</th>
<th>□ □ □ □ □ □</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which shape has the least?</td>
<td></td>
</tr>
<tr>
<td>Which shape has 2 more than the ○?</td>
<td></td>
</tr>
<tr>
<td>How many ○ and △ are there altogether?</td>
<td></td>
</tr>
</tbody>
</table>

**Collecting, sorting and representing data - tallies and bar graphs**

A bar graph is a graph made of vertical columns, each represents one category (e.g. type of fruit). Use graph or grid paper to draw your bar graph on, as this makes it much easier to keep your scale consistent without having to go to any effort measuring. As you can see, the bar graph looks similar in some ways to the pictograph discussed previously (pictographs show small icons that give a visual effect similar to columns or bars of different heights).

Here is an example of a bar graph to represent the data from a table.

<table>
<thead>
<tr>
<th>Occurrence of the numbers thrown with a die</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

**ACTIVITY 2**

Watch the video "Data Handling" (3:15 minutes) where a teacher collects data from her Grade 3 class by using tallies.

- Which parts of the Data Handling cycle are evident in the video?
- How does the teacher collect the data?
- How is the data organised?
- Is the data reported on and discussed in the video?
Commentary

Tally marks are a great way for learners to collect information. Many learners have a tendency to draw dots, and then become very confused when trying to count dots that are scattered all over the page. Tally marks teach learners to organise their data into a logical way, and it also helps them to count their data by skip counting in 5s. Remind learners that they need to draw 4 lines next to each other, and then the fifth line is a diagonal line through the other four to indicate a complete group of five.

In this video, we can see that three of the four parts of the data handling cycle are covered. The teacher collects data from the learners. The learners need to raise their hands and she counts the number of hands. The teacher then uses tally marks to organise the information. There is also an opportunity to discuss the tally marks, and to comment on the number of learners votes that they represent.

Interpreting data - pictographs and bar graphs

In the final part of the data handling cycle, learners are expected to analyse and interpret the data indicated on the graph. There is a difference between reading information off the graph, and actually interpreting the graph.

Reading information means that the information is clearly indicated on the graph, and that learners just need to look for it and read it off. Questions that require learners to read information from the graph would include:

- How many learners like oranges?
- On what day did the highest number of cars drive past the gate at the repair shop?
- Which is the least favourite fruit?

It is clear that these questions require an understanding of graphs, and that learners will need to know where to find the information. They will need to know that the x and y axes indicate...
different information, and that they must look for the answers in the correct place. However, if they do understand the layout of graphs, then answering these questions becomes relatively simple.

Interpreting questions is a little more challenging, as the information needed to answer the question is evident in the graph but it is not as explicit as it would be for the questions discussed above.

Interpretation usually involves explaining or giving reasons for an answer, not just stating an answer. Referring to the graphs shown above the teacher could ask questions such as:

- Why do you think more cars drove past the repair shop on the weekend?
- Why do you think so few learners like bananas?

Another example of this would be when the teacher asks questions relating to a class graph on favourite pets. On the graph, it is shown that there are 7 dogs, 5 cats, 3 hamsters and 0 snakes. The teacher could ask “What can you tell me about the snakes?” A learner who reads the graph may answer that there are 0 snakes on the graph. A learner who interprets the graph may respond by saying “Snakes are the least favourite. No one chose them because they are scary”. In this response, the learner has recognised that there are zero snakes on the graph because no one voted for them, and the learner gave a possible reason for this.

**ACTIVITY 3**

Watch the video “Tallies and bar graphs” (3:27 minutes) of a Grade 3 class using a bar graph to find out more about the favourite colours of the learners in the school.

- Which parts of the Data Handling cycle are evident in the video?
- How do learners interpret the data in the bar graph?
- How is the bar graph different and similar to the pictograph used in Lesson 9, Activity 1?

**Commentary**

In the video the teacher is working with a bar graph which represents the favourite colours of the learners in the school. She encourages the learners to interpret the graph by asking them questions about the represented data. She asks questions that get the learners to read the graph, where they can look at the axes of the graph and easily identify the number of learners who voted for a particular colour. The teacher also asks questions to encourage the learners think a bit more about the data. When she asks which colour T-shirt they should order, the answer is not immediately evident. The graph identifies favourite colours, so the learners need to think a bit about what this means in terms of buying T-shirts. They should be able to come to realisation that if blue is the most popular colour, then it would be logical to order blue T-shirts as most learners like that colour.
In both Lesson 9 Activity 1 and Lesson 10 Activity 3, the learners were looking at data regarding the favourite colours of learners in the school. Although the actual data collected is different, the purpose of the graph in both activities is to represent the favourite colours of learners in the school. In the activity in Lesson 9, a pictograph is used. As we have already discussed, a pictograph uses pictures to represent data. When representing colours, the teacher did not need to choose a picture as such, but she rather just used a block of colour. This means that the pictograph and the bar graph look quite similar, although there are clear differences. The pictograph has individual blocks of colour as ‘pictures’, whereas the bar graph has long bars of colour.

Check your understanding: True or False?

1) The pictures used in a pictograph can be unrelated to the data being represented.

2) A bar graph can be made of different pictures.

3) Tally marks can create confusion when learners try to organise their data.

4) Interpreting data requires some understanding of the information recorded on the graph.

REFLECTION

• Reflect on your experience of teaching Data handling – in what ways does data handling provide a context for the learning of other mathematical concepts?

• What other skills and concepts (in addition to mathematical concepts) do you think could be developed through data collection activities?

Well done you have completed Lesson 10.

Answers:

1. False - The pictures used in a pictograph usually relate in some way to the data being represented.

2. False - A bar graph is made of columns.

3. False - Tally marks are a good way of recording and organising data.

4. True