NUMERACY ACADEMY

PART 2:
ADDITIVE OPERATIONS
AND PATTERNS

TEACHING
CHILDREN MATHS

10 HOURS
Numeracy Academy

A team of writers from Bala Wande developed the Mathematics content of the Numeracy Academy drawing on the Bala Wande Thinking Maths modules, in consultation with Cally Khune of RED INK. The materials also draw on the Bala Wande Foundation Phase materials (Grades R to 3) were developed in consultation with a reference team of early Mathematics specialists.

www.fundawande.org
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Bala Wande Grade R Mathematics Programme Teacher and Teacher Assistant Training Facilitator Guides. (2021, 2022).

Bala Wande Grade R-3 Mathematics Programme Teacher Guides and Learner Activity Books. (2021, 2022, 2023).


Welcome and Orientation

Welcome to the Numeracy Academy – this course aims to get you to start Thinking Maths!

The materials are divided into 4 parts. Each part should take you 10 hours to complete.

It is important that you go through the lessons in sequence as each lesson builds on the content from the previous one.

We encourage you to be an active reader while engaging with each lesson.

Each lesson has video(s) that you need to watch by clicking watch now. If you are reading the print version of the booklet you can use the QR code to access the video.

Each lesson also has a self assessment that you should complete. This will give you a chance to recap on what you learned in the lesson.

Each lesson ends by providing you an opportunity to reflect on what you have learned in the lesson. Take time to do this activity - it will help you consolidate and action the learning in your classroom.

REFLECTION

- Reflect on your own school experience as a child. Did you have opportunities to play as a means of learning?
- Think about your own practice. Does your teaching provide opportunities for children to play? Or are your lessons mainly teacher-led? Write a personal goal in relation to enhancing the teaching and learning in your classroom by including playful experiences.
In order to gain the most from this course, please ensure that you watch the videos in full and that you complete each self-assessment. The assessments during the course are self-checks and the answers are given at the end of each lesson. As part of the final assessment of the module there will be two tests.

- Test 1 is taken after the completion of Part 1 and 2.
- Test 2 after the completion of Part 3 and 4.

- Each test lasts 1 hour and is in multiple-choice format.
- An online link for each test will be provided on the scheduled date.
- You will receive your results after clicking the submit button at the end of each test.
- If you fail the test you will be provided a second chance to take the test and a new date will be scheduled for this.

We hope you enjoy the course and find it beneficial!
In Part 2, you will be looking at addition and subtraction. We will discuss the importance of mathematical language, and we will investigate patterns, the use of resources and different strategies used in the operations. We will also learn more about using addition and subtraction in problems involving money. Finally, we will spend time exploring addition and subtraction as inverse operations.
In this lesson, we will introduce the concept of addition. We will begin by looking at Mathematical language, and its importance in the development of a sound number concept. We will then revisit the notion of play, considering how addition can be taught in context. We will also discuss assessment, and think about learner errors, so that we can further our understanding of how children learn.

What you will learn in this lesson

- The importance of Mathematical language
- Addition in context - learning through play
- Using assessment and learner errors to inform teaching

Mathematical language

It is important to provide multiple opportunities for children to be exposed to mathematical language. The knowledge and understanding of appropriate mathematical vocabulary helps children to express their thoughts and ideas about mathematical concepts. This ability to use mathematical language accurately is an essential part of the learning process. Children will use this language to participate in and complete tasks, and to respond to questions.

In some instances, there is a difference between a child’s social (or everyday) understanding of a word and his academic (or mathematical) understanding of a word. An example of this would be the word ‘table’. In a social context, the word describes a wooden piece of furniture at which we might sit to do our homework. In an academic context, the word refers to the way in which information can be organized and displayed. Teachers need to help children to make connections between their prior knowledge and the new mathematical language. One way in which to do this is to focus on the use of resources as a way to introduce new concepts. Children need to be encouraged to use language to make connections between concrete apparatus, pictorial representations and abstract mathematical symbols. As a teacher, it is necessary for you to create a safe learning environment and to model the correct use of mathematical language so that children develop their ability to verbalise their solution methods.
ACTIVITY 1

Watch the video “How many altogether” (2:03 minutes) to see how the teacher teaches the concept of addition to her Grade 1 class.

• What specific mathematical language can you identify in the video?
• Write a list of any other words that you can think of that relate to the concept of addition.

Commentary

There are many words relating to the concept of addition that you may have identified: Add, addition, plus, equals, altogether, more than, increase, total, groups of, sum.

It is important to ensure that children recognise these words, and that they use them correctly. In our everyday language, we have developed habits of using words such as ‘sum’ to refer to any mathematical problem. This is not the correct use of the word, as in a mathematical context the ‘sum’ of a number refers specifically to the total amount resulting from the addition of two or more quantities. Therefore, when we use the word ‘sum’ to refer to a subtraction problem, we are telling children that the vocabulary used in mathematical contexts can be used interchangeably. We cannot then be surprised when children make statements such as my half is bigger than your half. Children need to understand that the words used in maths have specific meanings, and they should use them appropriately. This will prevent confusion as they progress throughout their schooling and help them to respond to questions accurately. Ultimately, a sound understanding of mathematical language is the first step to improving learner performance in mathematics.

Addition in context

When we introduce any operation, we should begin by using numbers in a real context. To do this, we can tell stories that lead to the addition or subtraction of numbers. This makes it clear to the learners what they need to do, and it also lays a foundation for their own problem-solving later, when they will have to read and interpret word problems. These stories help children to see mathematics as being relevant to their everyday lives, rather than something that is only done in a Maths lesson at school.

In Maths, we refer to children’s ability to solve problems effectively as mathematical fluency. This means that children are able to flexibly select the best strategy to help them solve problems quickly and accurately. In order to improve children’s mathematical fluency, we need to allow them time to learn number facts, such as their number bonds, until they are able to recall these with ease. However, this is not just rote learning of number facts. Encouraging children to verbalise their methods helps them to develop their conceptual understanding, which in turn will enable to face problem-solving tasks with greater confidence.
ACTIVITY 2
• Give an example of an addition story that would be relevant to the children in your class.

Commentary
Think about what the children in your class may be interested in, such as marbles, sweets or balloons. You may even be able to include items related to the topic you are covering in class at the moment. An addition story needs to be simple and clear, so that children can work out what needs to be done to solve the problem. Initially you may even keep to the same sentence structure, such as:

Nosipho has 5 balloons.
Her brother gives her 3 more balloons.
How many balloons does she have now?

Children need to learn to identify the important information in the addition story, and to use this to solve the problem. Once they have become confident in this, you could increase the complexity of the stories.

Learner Errors
In order to help children become confident in their mathematical fluency, and to effectively reason and solve problems, we need to continuously assess their learning and understanding. Some assessments may take the form of written tasks; however, it is more meaningful to assess through continuous observation, and a combination of practical and oral tasks as well. If we rely on just one written task to assess children, then we do not take in consideration any contextual factors that may influence a child’s performance. It is important to remember that assessment is an emotional process, and that many children experience anxiety over formal assessment tasks.

To provide rich opportunities for observation in class, give your learners time to play games and while you do so watch the way they they:
• use mathematical language
• give mathematical answers
• deal with errors they or their partners in the game make.
It is important to use your assessment to improve the teaching and learning that takes place in your class. Through careful observation and discussion with the children, it is possible to determine where their misconceptions may fall. You can then use this knowledge to redirect your future lessons, and to rectify the children's misconceptions.

**ACTIVITY 3**

Look at the problem below.

\[
30 + 500 + 7 = \square
\]

- What do you think the child's misconception might be?
- What could you do to rectify this misconception?

**Commentary**

In this example, the child has added the digits without considering their value. He has disregarded the zeros, assuming a nil value for those, and then added the 3, 5 and 7 as if they were all ones. The child does not have a sound understanding of place value, and additional work needs to be done to help him to move on from viewing numbers as a collection of ones to a system of tens. Concrete resources such as base ten blocks can be used to help him recognise that 10 ones is the same as 1 ten, and that 10 tens is the same as 1 hundred.
Check your understanding: True or False?

1) The word ‘sum’ refers to all mathematical problems given to children.

2) Stories are used to make Maths more entertaining for young children.

3) Written assessment tasks are the most valuable way to assess children.

4) Mathematical language plays an important role in the assessment of children as well as in rectifying misconceptions.

REFLECTION

• Reflect on your use of mathematical language in the classroom.
• How could you model the correct use of specific vocabulary?
• Write a personal goal in relation to the promotion of mathematical language in your classroom.

Well done you have completed Lesson 1.
In this lesson, we will focus on patterns. Patterns are part of our everyday lives, and it is human nature to seek out and identify patterns. We will look at types of patterns, and then consider how patterns are used in the teaching and learning of addition. We will also discuss number lines and the various strategies that children can use when solving addition problems.

**What you will learn in this lesson**

- Why are patterns important?
- Patterns and number lines
- Addition strategies

**Why are patterns important?**

Patterns help us to simplify our world by classifying different objects and situations, allowing us to create a sense of order into our complex environment. The ability to recognise the same pattern in different situations enables us to transfer our knowledge of one context to another context without studying each one separately. Order, regularity and sequence are at the heart of Mathematics, and children need to develop their ability to find and recognise patterns.

Children can be exposed to patterns in everyday life, such as the patterns found in curtains, tiles, windows and bricks. Children can also physically experience patterns by playing hopscotch, clapping games, and by identifying visual, auditory and tactile patterns.

Patterns play a valuable role in the teaching and learning of number concept. Unless children can recognize and apply number patterns, their only tool for problem solving is counting. So, if we want to help children progress from solving problems by counting all, we need to help them identify numerical patterns that will improve their mathematical fluency. The images show a teacher teaching about patterns using number bonds. In the second image the teacher has removed the bonds of 7 and is discussing the pattern made by doing so.
ACTIVITY 1
Think about the patterns you encounter in your everyday life.
• Describe a visual pattern that you can see in your classroom.
• Describe an auditory pattern that you may have heard recently.
• Describe a tactile pattern that you could get the children in your class to experience.

Commentary
Activities where children discover patterns, and the rules which are part of these patterns, lay a foundation for algebra in later schooling. Getting children to identify a variety of patterns, and to talk about the patterns, helps them to recognise the rules that they have created in order to establish the pattern. Even when the patterns may be auditory or tactile in nature, or comprised of colours, shapes, different sized objects or any other visual possibility, the identification of the pattern lays the foundation for learning about number patterns. Once children are able to recognise and describe visual, auditory and tactile patterns, they will then be able to transfer these skills to activities involving number patterns. Pattern recognition is part of mathematical reasoning. Activities that involve deciding whether or not numbers fall into the same set call on this pattern recognition skill.

Patterns and number lines
Teachers can continue using patterns to help children develop their mathematical fluency when they focus on the use of number lines. Number lines are useful as they show the position of numbers in relation to one another. This gives children opportunities to verbalise their understanding of number, using the appropriate mathematical vocabulary. Number lines can also be used to promote ordinal numbers and counting skills. Children can use a number line to count forwards and backwards, in ones or in multiples, as well as counting on or back from a given number. Children can discuss the patterns that they discover as they use number lines to addition and subtraction problems.

Young children can start to experience placing numbers in sequence on a number line using number cards as shown in the following images. When doing such an activity the teacher should ask questions about the relative sizes of the numbers and how to make the decision of what number is placed where on the number line.

When children are able to use patterns to help them solve problems, this means that they are able to rely on previously constructed knowledge and understanding in the process of working out their solutions. The fact that they do not need to re-construct their understanding for each new problem means that they are working with greater efficiency. As previously mentioned, our goal as mathematics teachers is to help children to flexibly select the best strategy to help them solve problems quickly and accurately. If children are able to do this, using a strong foundation of mathematical language, they will be well on their way to confidently approaching tasks involving reasoning and problem-solving.
ACTIVITY 2

Number Lines and Number Patterns

Watch the video “Number Lines and Number Patterns” (4:40 minutes) to see how the teacher uses the idea of patterns to help children count in 2s on a number line.

- Describe the patterns identified by the children in the video.
- Can you see another pattern that the children didn’t identify?
- How would you help children to see the patterns involved in counting in 2s on the number line?

Commentary

By identifying the patterns involved in counting in 2s on the number line, the children are creating a foundation for the idea of repeated addition. They can see that the jumps on the number line are the same as adding 2 each time. The teacher also discusses the idea that there is pattern of coloured buttons. It is possible to elaborate on this pattern, in that the order of the three colours is reversed for the second group of three buttons. A pattern that was not identified in the video was that the placement of the buttons skips one number each time. This could also have led to a discussion on odd and even numbers.

The teacher tries to model the use of mathematical language, by rephrasing the answers given to her by the children. This is an important part of the process, as children need a great deal of practice in order to be able to verbalise their ideas confidently. When the teacher repeats or rephrases what the children have said, she is showing them how to express themselves clearly, using the appropriate terminology.

Addition strategies

As part of the continuous assessment process, teachers can use observation, practical activities and oral tasks to help them to recognise some of the strategies that children use to solve problems. Many children struggle to make the shift from solving problems by counting to using calculation strategies. It is therefore essential that teachers spend time drawing children’s attention to the different strategies and to model these in their teaching. Teachers should be aware of the range of available methods, so that they can support the children who use these various methods. Children can use the appropriate mathematical language to discuss and share their methods, and this can encourage other children to try out the methods. With practice, children will then begin to select appropriate strategies for the problems they are given.
Children know that 7 + 7 is 14 and they take away 2 from 14.

8 + 8
children will know that the answer is 16 because they know that 8 doubled is 16.

9 + 6
add 1 to 9 in order to have 10 and add 6 to 10, subsequently, they subtract 1 from 16. The answer is 15.

8 + 7
take 2 from 7 and add it to 8 in order to make 10 and add 5 to 10. The answer is 15.

**ACTIVITY 3**
Look at the addition strategies discussed above.

- Select one of the strategies and describe the pattern that is evident in the use of this strategy.
- How could you help children identify the patterns in addition strategies?

**Commentary**
When children move on to using calculation strategies rather than simply counting to solve problems, it is helpful highlight the patterns involved in these strategies. There is a tendency to teach calculation strategies as a method to be learnt by children. However, this tends to make children try to follow a ‘recipe’, which often results in errors because they do not fully grasp the strategy that they are using. If children are given opportunities to discover calculation strategies for themselves, and to verbalise the patterns involved in using these strategies, they are more likely to understand the process.
Check your understanding: Multiple Choice

1) Patterns:
   a) Create a sense of order.
   b) Are pretty to look at.
   c) Both of the above.

2) Pattern recognition:
   a) Is only important for pre-number concepts.
   b) Is a transferable skill.
   c) Cannot be taught.

3) Recognising patterns in number lines and addition strategies:
   a) Does not help children solve problems more efficiently.
   b) Shows children the recipe for solving problems.
   c) Helps children to understand what they are doing when they choose strategies to solve problems.

4) It is important to talk about patterns because:
   a) It gives children an opportunity to be creative.
   b) Children learn to verbalise their reasoning as they explain the rules of their pattern.
   c) Children need to be able to speak confidently in class.

REFLECTION

• Reflect on your experience of patterns in your everyday life.
• Think about whether or not you have explicitly addressed the idea of patterns in your class.
• Describe what you could do to increase the opportunities for pattern recognition in your lessons.

Well done you have completed Lesson 2.
In this lesson, we will look at the use of manipulatives in the solving of addition problems. We will first discuss the different types of manipulatives that could be used, and then consider the progression involved in the use of resources. Finally, we will investigate the use of ten frames in addition problems.

What you will learn in this lesson

• What are manipulatives?
• Addition and concrete resources
• Addition and ten frames

What are manipulatives?

Manipulatives are items used by teachers and children to represent abstract mathematical concepts. Research tells us that children use manipulatives to bridge the gap to an abstract understanding of number and mathematical operations. By physically handling concrete resources, children begin to make connections between the mathematical language, symbols, and pictorial representations. These connections lead children towards the development of mental images which will help them to solve problems in a more abstract way.
ACTIVITY 1
For each apparatus mentioned give one practical way in which that apparatus could be used in the teaching of addition.

1. Abacus
2. Base ten blocks and ten frames
3. Multifix cubes
4. Hundred squares
5. Number lines and number tracks

Commentary
An abacus is a useful way for learners to manipulate the beads and to observe the groups of five and ten. This helps them to develop a sense of these numbers as ‘anchor’ number, which then helps them to develop efficient strategies for solving problems. Base ten blocks can be used for simple addition of small numbers, as well as for addition of up to three-digit numbers. Multifix cubes are plastic blocks that can be stuck together and taken apart, and so these are useful to use in a variety of activities. Children can make groups of numbers, they can add cubes to represent an addition problem, and they can also make towers of ten to represent place value. Hundred squares can be used for counting on, as well as for demonstrations of bigger number addition and subtraction, using accelerated counting on or taking away. Number lines are a valuable resource to show jumps along a number line for early addition problems. Bigger jumps can also be shown on a number line, helping children to count in multiples. The use of number lines is very good in consolidating number concept.

Addition and concrete resources
As we have seen from our discussion on manipulatives, it is important for children to be introduced to a concept with the aid of concrete resources. When learning about addition, children can use a number of different items such as counters, cubes and beans. These resources provide a visual representation of number, helping children to establish connections between the abstract symbol, the mathematical language and the physical quantity. The resources will also enable the children to discover the notions of more than and less than as they physically position the items.

Once children have actively constructed their understanding of addition by using concrete resources, then they can progress to pictorial representations. Some children may need to begin by using the concrete resources, and then drawing these so that they have both representations visible at once. This will help them to make the shift from physical objects to the more abstract two-dimensional drawings. Other children may find that they can progress on to the pictorial representations without needing to see the drawings next to the concrete resources.

Once the children are confident with the use of pictures to represent addition problems, then they can progress to the abstract symbolic representation. It is important that teachers don’t move children on to this stage before they are ready.

ACTIVITY 2
Describe activities in which you can help children to develop their understanding of the bonds of ten.

- In the first activity, use concrete resources to help children.
- In the second activity, use a pictorial representation to represent the number bonds.
- In the third activity, focus on the symbolic or abstract representation of number bonds.
Commentary

There are many possibilities you could use as concrete resources when teaching children about the bonds of ten. One option is the number mat that was discussed in Part 1 Lesson 6. Children could take 10 counters and scatter them on the number mat. They would then record how many counters fell on the left-hand side of the line, and how many counters fell on the right-hand side of the line. They could record this in their book, before collecting up the counters and scattering them again. In doing this, the children will discover the different bonds of ten and create the different number bonds.

Then, the children could draw dots as a pictorial representation of the bonds of ten. This is a super way of helping children identify the pattern of the number bonds, and to create a mental picture for themselves.

This can then be further extended into the number bond ‘house’, which is a symbolic representation of the number bonds. Once again, this is useful for children to see the pattern of the bonds.

Addition and ten frames

As children develop their sense of number, they can learn to subitise by using ten frames. The layout of the ten frames makes it easy for children to see five or ten quickly. Children will also begin to determine numbers like 7, 8 or 9 more easily as they will recognise that the empty blocks on the ten frames help them to work out the number. A particular point to note is the fact that children may use addition or subtraction to work out missing numbers on a ten frame. For example, children may see 6 counters on the ten frame and count on to 10. They could also identify that they know that 10 - 6 = 4, so therefore the empty blocks equal 4.

It can easily be seen that ten frames are a particularly worthwhile concrete resource to use when learning about addition. They provide an opportunity for children to physically move counters on the frame, creating a visual representation of the bonds of ten. Children can verbalise what they see as they construct their understanding of the bonds of ten. By placing counters on a ten frame, it is easy to see how many more counters are needed to make ten.

ACTIVITY 3

Addition using Ten Frames

Watch the video “Addition using Ten Frames” (5:20 minutes) to see how the children use ten frames to help them solve addition problems.

• What did you notice about the way the first child positioned the counters on the ten frame?
• Why do you think the way the second child positioned the counters is better?
Commentary

In the video, the first child laid out the counters as shown below:

![Image 1](image1.png)  ![Image 2](image2.png)

The second child then adjusted the layout of the counters to look like the example shown below:

![Image 3](image3.png)  ![Image 4](image4.png)

Whilst this is not incorrect, it is easier to see the number 15 if the counters are positioned as the teacher demonstrated. This ties back to our discussions on subitising in Part 1, where we discovered that the layout of objects helps us to quickly determine a number without counting. With the full ten frame on the left, we can see this as a complete ten, and it lays the foundation for our work with place value tables where the tens are on the left and the ones are on the right.

Check your understanding: True or False

1) Concrete apparatus helps children create connections that will help them to work in a more abstract way.

2) Children’s understanding of concepts moves through a progression from concrete to symbolic to pictorial.

3) Number bonds are number facts that just need to be learnt off by heart with no need for manipulatives.

4) There are multiple ways to lay counters out on a ten frame, but some ways make it easier to work out the number.

REFLECTION

- Reflect on your use of manipulatives in your own classroom.
- What do you think the main challenges are / would be in terms of using manipulatives in your lessons?
- What do you think the main benefits are / would be in terms of using manipulatives in your lessons?

Well done you have completed Lesson 3.
In this lesson, we will continue investigating the use of manipulatives in the solving of addition problems. When working with larger numbers in addition, base ten blocks are a useful resource to help children understand the value of the numbers. However, as children become more confident in the solving of problems, a move towards using pictorial representations becomes an option. We will discuss both of these situations, before looking at some of the potential problems with using manipulatives in the classroom.

What you will learn in this lesson

• Addition and base ten blocks
• Pictorial representations
• Potential issues with manipulatives

Give learners lots of time to become familiar with the manipulatives you will use. Base ten blocks are particularly useful in the teaching of 3-digit number sense. Flard cards are also useful. They can also be used when teaching addition and subtraction of 2- and 3-digit numbers. Learners need to develop a sound number sense (discussed in Part 1) - this will begin when you introduce them to the numbers and it will be consolidated when they work carefully with numbers in the context of additive operations. It is always a good idea to allow learners time to play conversational games where they can develop their mathematical language. They can spend time showing and talking about numbers using manipulatives in preparation for using the manipulatives when doing operations.

Addition and base ten blocks

When adding larger numbers, children can use base ten blocks with a place value mat as concrete manipulatives. This is a useful way for children to physically see the process of equal exchange. They can lay out the tens and the ones on to the place value mat, and then add them together. In the images below, we can see that the children add 5 ones and 2 ones first. They then add 2 tens and 4 tens and 1 hundred to get a total of 167.
Once children have consolidated their understanding of adding larger numbers without regrouping, they will be able to use base ten blocks to solve problems involving regrouping.

Correct! We can exchange and make 1 hundred. How much do you have altogether?

I have 1 hundred, 2 tens and 9 ones.

ACTIVITY 1

Higher Number Addition using manipulatives

Watch the video “Higher Number Addition using manipulatives” (5:20 minutes) and see how the teacher encourages the children to use their base ten blocks.

- How do the children initially solve the problem 23 + 39?
- What is the ‘quick counting method’ that the teacher encourages the children to use?
- How does the use of base ten blocks help the children to develop their understanding of place value?
Commentary

In the video, the children need to solve an addition problem that involves regrouping. In order to solve the problem, the children are encouraged to use the ‘quick counting method’. This means that they exchange ten ones for one ten. The teacher is trying to get the children to practise equal exchange so that they can consolidate their understanding of place value. It is important for children to recognise that they can put ten ones together to make one ten, and that it is easier and quicker to count in tens than in ones.

Children need to have many opportunities to practice equal exchange. This is a difficult concept for children, and the use of physical resources help them to develop a better understanding. The use of manipulatives prepares children by showing them what exchange actually involves before they attempt to solve problems in an abstract way.

Pictorial representations

As children practice addition with larger numbers, it will become necessary for them to start drawing the representations of the numbers rather than try to manage increasing quantities of physical apparatus. This is a logical progression in their understanding and will further prepare them for the more abstract methods of solving problems.

Some children may need to have the concrete apparatus in front of them before drawing the resources, whilst others may be able to make the shift from concrete to pictorial more effortlessly. It is important for children to develop a quick and easy method of creating pictorial representations. Initially, children may try to draw perfect representations of the resources, but they need to learn to draw simple shapes or lines so that they can solve problems quickly.

A number line is another pictorial representation that is often used to represent numbers. To draw a number line correctly you need to choose an appropriate scale and your measurements must be accurate.

ACTIVITY 2

Look at the number line and addition problems below.

• How could you help children to use the number line to solve the problems?
• Why do you think a number line is counted as a pictorial representation?
Commentary

The children can draw jumps on the number line to show their addition. Initially they may be counting on from the first number, using a simple method of solving the problem. As their understanding of number progresses, they will begin to select other calculation strategies such as compensation or bridging the ten. If children were given the problem 138 + 6, they may choose to break up the 6 into 2 + 4 so that they could solve the problem as shown below:

In this way, the children can add 2 to get to 140, and then add on 4 to complete the calculation. They would do this because it is easier to use the multiples of ten as pivot numbers, rather than trying do equal exchange mentally.

Some potential issues with manipulatives

Many children find it difficult to move from the concrete representations of number to more abstract symbolic problems. This may be due to the fact that children become dependent on the concrete resource and rely heavily on the physical action involved in manipulating the items. Teachers may also underestimate children’s ability to solve problems in an abstract way, and so forget to encourage them to move beyond using the resources when appropriate.

Researchers believe that mathematical language is the key to bridging the gap between using concrete resources to solve problems and a more abstract method of solution. Children need to be encouraged to verbalise their actions as they manipulate the resources, and to consolidate their understanding of the concepts. By doing this, they are constructing mental images which can then replace the need for the physical resources.

As children begin working with larger numbers, it becomes even more necessary for them to shift to an abstract method of solving problems. Trying to use concrete resources to solve problems involving four-digit numbers can become problematic. At times, the number of resources required to construct large numbers becomes overwhelming, and results in more errors as items become misplaced or overlooked.

ACTIVITY 3

Look at the problem below:

345 + 587

• What resources do you think children should use to solve this problem?
• Why did you choose this resource rather than any other?

Commentary

When solving problems that involve large numbers, it becomes impractical to use certain concrete apparatus. For example, to use counters or multifix blocks for this problem would be difficult as there would be too many resources for the children to count or sort accurately. Base ten blocks would be more manageable as children would be able perform an equal exchange, and physically show the total amount. Children may also use a number line, as this would be an efficient method of solving a problem with large numbers. They could show jumps along the number line, from a simple method of counting on to the more advanced calculation strategies.
### Check your understanding: Multiple Choice

<table>
<thead>
<tr>
<th>Question</th>
<th>Options</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>1) Base ten blocks are:</td>
<td>A) Useful for working with larger numbers.</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B) Problematic when solving regrouping problems.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C) Both A and B</td>
<td></td>
</tr>
<tr>
<td>2) It is important for children to:</td>
<td>A) Count in ones when solving problems.</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>B) Solve problems without physically handling the base ten blocks.</td>
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<td></td>
<td>C) Practise equal exchange by using concrete apparatus.</td>
<td></td>
</tr>
<tr>
<td>3) Pictorial representations are:</td>
<td>A) An unnecessary step in the progression of learning.</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>B) Easier to manage than large quantities of concrete resources.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C) Time consuming.</td>
<td></td>
</tr>
<tr>
<td>4) Mathematical language is:</td>
<td>A) The bridge between using concrete resources and solving problems abstractly.</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B) Unimportant in the progression of children's learning.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C) The bridge between confusion and understanding.</td>
<td></td>
</tr>
</tbody>
</table>

### REFLECTION

- Think about what you have noticed in terms of children's progression of learning in Maths.
- What have you noticed about children's ability to verbalise their actions and their level of understanding?
- What have you noticed about the amount concrete and pictorial resources used in your lessons?

Well done you have completed Lesson 4.

### Answers

1) A, 2) C, 3) B, 4) A
In this lesson, we will discuss Money. This is an important topic, and children need much practice in order to confidently solve money problems. As teachers, we are often worried about how much time we have available to get through the curriculum, and so we focus more on getting children to solve money problems rather than on coin and note recognition. However, children will find money problems and practical activities extremely difficult if they are unsure about the notes and coins, so the recognition of these should be our starting point in this topic.

What you will learn in this lesson

- Coin and note recognition
- Addition and subtraction with money
- Practical activities

Coin and note recognition

When looking at coins and notes with your children, it is a good idea to make sure that children are actively involved by looking at resources, and describing what they see. If you just tell them what is on the coins and notes, they are likely to forget easily, whereas if they investigate for themselves, they are more likely to remember what they have learnt. The South African Reserve Bank and the South African Mint are both fantastic sources of information if you would like to find out more facts about the notes and coins we use.

In 2023, the South African Reserve Bank has revised the coins and notes in circulation and so there are quite a few changes to look out for. The 1c, 2c and 5c coins have not been in circulation for some time now, leaving only the 10c, 20, 50c, R1, R2 and R5 coins. The R5 coin now has a blue whale on it, the R1 coin has a protea, the 50c has a Knysna Turaco, and the 10c coin has a bee on it, which is the first time an insect has been represented on a South African coin.

The notes have been adapted to included increased security features, so that people can use the Look, Feel, Tilt strategy to check if the notes are fake. When you look at the notes, you can see the watermark and the numerals. There are certain features which are raised on the notes so that you can feel them, and these particularly help the blind and partially blind communities. When you tilt the notes, you will see a circle that spins and changes colour.

ACTIVITY 1

Research the South African coins and notes, and then brainstorm activities that you could use to help children recognise them.
Commentary

<table>
<thead>
<tr>
<th>Money Bingo</th>
<th>Children can play a bingo game where they identify the different coins or notes called out by the teacher.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have... who has... ?</td>
<td>Children get cards that say (for example): ‘I have R10, who has 50c?’ The person with the card that says ‘I have 50c ...’ would then have to read their card aloud. Pictorial representations of the notes and coins would help children recognise their features.</td>
</tr>
<tr>
<td>Snap game</td>
<td>Children could get cards with money amounts written on them, and then cards with coins or notes on them. Children could play snap where they match the cards.</td>
</tr>
<tr>
<td>Memory game</td>
<td>This would be much the same as the snap game, but the children would have to try recall the location of the cards that are placed face down on the desk.</td>
</tr>
<tr>
<td>Quiz</td>
<td>The teacher could call out features and children could hold up the correct coin or note that matches that feature.</td>
</tr>
</tbody>
</table>

Addition and subtraction with money

In the beginning, when first starting addition and subtraction with money, it is sensible to reinforce the idea of more than, less than and the same amount. You can hold up a coin, and ask Who has more?, Who has less? or Who has the same amount as me?. Children will need to recognise the value of their coins and hold up their coins in response to your questions.

Children also need to realise that an amount of money can be made up in different ways.

This is a challenging concept, and children will need multiple opportunities to consolidate their understanding. It is a worthwhile idea to gather a collection of priced items from supermarket specials leaflets. You can then use these to ask questions such as Show me how you will pay for this? and Who else paid in this way?. This gives children a chance to verbalise their selection of coins or notes, and to listen to the selections made by their friends.

As you can see from the videos shown, there are many opportunities to solve addition and subtraction problems using notes and coins. You could use the notes and coins in the place of base ten blocks, using these to practice equal exchange.
ACTIVITY 2

Money Matters (addition)

Watch the video "Money Matters (addition)" (5:37 minutes) to see how children solve problems using notes and coins.

- Explain how the children use the notes and coins to solve the problem.
- How is this method similar to using base ten blocks as a resource?
- What does the video highlight about counting methods?

Commentary

The children use R10,00 notes and R1,00 coins in much the same way as they would use base ten blocks to represent tens and ones. They are able to use the notes and coins to add two quantities together, practicing equal exchange when they notice that they have eleven ones. The teacher encourages them to use mathematical language to verbalise their exchange, as they replace ten R1,00 coins with one R10,00 note.

It is evident that the children count in ones as they solve the problem. It is essential that we encourage children to move beyond counting in ones in order to solve problems more efficiently. As previously discussed in both Part 1 and earlier in Part 2, we need to model the different ways of getting to an answer. As teachers, we need to help children get used to counting on from the larger number, using bonds of ten and skip counting where appropriate.

Practical activities with money

Many teachers enjoy the opportunity to involve children in the transactions of buying, selling and the giving of change. This can be done by setting up a real-life shop in your classroom and bringing in items for the children to pretend to buy and sell. However, it is necessary to think about whether this is the most sensible option for you and your classroom. Setting up a real-life shop takes up a great deal of space and resources, which may be tricky to cope with in a classroom setting. It may be a better idea to use the priced items mentioned earlier in this lesson instead of real items. The pictures of items are easy to collect, and they take up less room in your classroom. You could use these pictures in a variety of ways:

- You could have loose pictures which children could select for themselves and place in a shopping trolley.

- You could paste a selection of pictures onto cardboard as a leaflet or a ‘shop window’, and then ask the children questions such as You have R10. Which items would you buy for your breakfast?

These are great ways to get children working with money in context. Children will need to think practically about how much money they have, and how much they can spend. It is possible to include an extra element by getting children to work out what change they will be given as well.
ACTIVITY 3
Describe an activity that you could use to get children to practise adding and subtraction with money.

- List the mathematical language that would be appropriate for this activity.
- Clarify the resources that you would use in this activity.

Commentary
For this activity, you might choose to use priced items, and tell children that they can go to the tuck shop to buy some items. You could have a collection of items to be bought, and you could ask questions to get the children thinking about how much they could afford to buy, and how much change they would receive.

You have R10,00

At the tuck shop you buy:

_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________

You spent: _________________________
Your change was: __________________

The mathematical vocabulary that might be relevant for this activity is: buy, spend, change, cost, afford, expensive, cheap, bargain, half price, discount. There are many options that you could include to reinforce your mathematical concepts. For example, you might add a sign to your 'shop window' that says, Half Price Sale or 3 for the price of 2! These additions would allow the children to practice halving or doubling within the context of the money problems.
Check your understanding: True or False

1) Many of the South African coins and notes were revised in 2023.
2) Coin and note recognition is an essential first step in learning about Money.
3) Coins and notes should only be dealt with in word problems.
4) Mathematical language is not important in the topic Money.

REFLECTION

• Reflect on your experience of teaching Money.
• Think about the kinds of practical activities you have / have not done in your classroom.
• Write a personal goal in relation to using resources to teaching Money.

Well done you have completed Lesson 5.
In this lesson, we will return to our consideration of mathematical language as we look at the concept of subtraction. Once again, we will think about the role language plays as children verbalise their solution of subtraction stories. Finally, we will discuss mental maths and how to actively involve children in oral activities.

What you will learn in this lesson

- Mathematical language
- Subtraction in context
- Mental Maths

Mathematical Language

As we discussed in Lesson 1 of Part 2, children need many opportunities to be exposed to mathematical language. As we discuss the concept of subtraction, it is necessary to highlight again the idea that mathematical vocabulary helps children to express their thoughts and ideas about mathematical concepts. In subtraction, there are certain key words or phrases that are essential to children’s ability to solve problems correctly. For example, the word ‘difference’ signifies the answer that is given when one amount is subtracted from another amount. So, for example, in the number sentence $7 - 2 = 5$, we would say that five is the difference. This is a key point of understanding for children if they are to solve word problems accurately. They may come across a word problem such as:

For a child who does not understand the significance of the word ‘difference’, this becomes a challenging question because they cannot work out what they are meant to do. This question is trying to get children to work out how many numbers lie between 6 and 10. They could solve the problem by counting on from 6 to 10, or back from 10 to 6 on a number line.

Children may also solve the problem by working out $10 - 6 = \square$, or even by recalling their number bonds and saying $6 + \square = 10$. It is clear that children will need a great deal of practice with mathematical language in order to successfully identify the key information in the question.

ACTIVITY 1

Brainstorm activities in which you can help children develop their use of the mathematical language related to subtraction.
Commentary

It is a good idea to try and encourage children’s use of mathematical language in fun ways. It is also necessary to remember that you need to ensure you have opportunities to show both language modelled by the teacher, and language being used by the children. Children need to hear examples of how the language should be used correctly, so you could play a game where you call out number sentences, using the different vocabulary, and children need to hold up number cards to show the answer as quickly as possible. You could add an extra element by calling out some addition problems, and children need to recognise that these are not subtraction and therefore not solve the problem. In a similar way to the game Simon Says, anyone who solves the addition problem, would then be out of the game. In addition to this, you also need to hear children using the language themselves as they verbalise their understanding so that you can determine their level of understanding.

Some of the words that you would need to identify include subtract, minus, take away, decreased by, fewer, difference, have left, left over, less than, remain.

Subtraction in Context

The concept of subtraction involves taking a given amount away from another given amount, to find out the difference between the two amounts. You should notice that subtraction ‘undoes’ what addition ‘does’. Because of this relationship between the two operations, they are known as inverse operations.

As we discovered when we looked at addition in context, we should start teaching subtraction by using numbers in a real context. This means that we will use subtraction stories to create a context for the problems that children will solve.

When we get children to solve subtraction problems, we need to encourage them to show all the working that they do using concrete apparatus. We can use a variety of manipulatives such as counters, base ten blocks, ten frames or even 2-D resources such as number lines, drawings or even paper pictures.

As with addition, children need to develop their mathematical fluency in order to solve problems quickly and efficiently. In addition to this, children will work on their Problem-Solving and reasoning skills. The subtraction stories are a helpful here, as children learn to identify the key information in the story, and then to use mathematical language to verbalise their solution strategy.

ACTIVITY 2

Give an example of a subtraction story that would be relevant to the children in your class.

Commentary

Identify items that the children in your class may be interested in, or that may relate to the topic you are covering in class at the moment. Ensure that your subtraction story is simple and clear, so that children can work out what needs to be done to solve the problem.

Initially you might choose to keep to the same sentence structure, such as:

- Sandile has 10 marbles.
- He loses 3 marbles.
- How many marbles does he have left?
As they did with their addition stories, children will need to identify the important information in the subtraction story, and to use this to solve the problem. Once they have become confident in this, you can increase the complexity of the stories.

**Mental Maths**

Mental Maths is usually the first part of our mathematics lessons, and it is where we develop the children’s mathematical fluency. As we get children to practice solving problems in their heads, we are consolidating their number sense and reinforcing many mathematical concepts. Another benefit of Mental Maths is that it improves children’s estimation skills and increases their speed in doing all Maths problems. This is because children learn to look at a problem and estimate what the approximate answer will be. They are then able to determine the best strategy to accurately calculate the answer quickly. Another benefit of improving children’s estimation skills, is that they learn to judge the reasonableness of their answers. This is an important mathematical skill that helps them to check their work.

Mental Maths facilitates a shift from concrete strategies to use more abstract strategies, encouraging children use their knowledge of number facts to perform calculations. In Mental Maths, children also learn to select the best strategy for each problem. They discover that certain strategies take a long time and may not easily allow them to find the correct answer.

Once again, it is necessary for teachers to model the strategies that could be used in Mental Maths, so that children see different methods that could have been used. Teachers also need to encourage children to share their methods so that they have the opportunity to verbalise their understanding. Children often respond differently to the explanation of a strategy given by another child, as opposed to the explanation given by the teacher. It is useful to allow these opportunities as some children may become motivated to try different strategies when they see what their friends have done.

**ACTIVITY 3**

**Game - Fast Maths with cards - 2 more and 2 less**

Watch the video “Game - Fast Maths with cards - 2 more and 2 less” (2:43 minutes) to see how you can play a Mental Maths game with your children.

- What resources do the children have in front of them?
- How do the children solve the problems?
- What do you think will happen if children play this game regularly?
Commentary

In this game, the children have to shuffle their cards, and then turn over the top card. They then need to either add or subtract 2 from the card that is shown. They do not use any manipulatives to help them solve the problem. The idea is that they need to solve the problem quickly in their heads, so that they can continue playing.

It is clear that at least one of the children is a little unsure, and so is hesitant to suggest answers. If children play this type of game regularly, then they become more confident in recalling their number facts, and the pace of the game will speed up. It is essential that children know their number facts well, and that they can recall these quickly and easily without having to count or calculate. If they are able to do this, then they are able to focus on solving problems without trying to work out the number facts first.

Check your understanding: Multiple Choice

1) An understanding of mathematical vocabulary:
   A) Will develop in later years of school.
   B) Helps children understand the question.
   C) Does not affect children's ability to answer questions.

2) Subtraction stories:
   A) Make Maths more fun.
   B) Try to trick children.
   C) Help children solve problems in context.

3) In subtraction stories:
   A) Keeping the same sentence structure helps children identify the key information.
   B) Use situations that are relevant to the children.
   C) Both A and B.

4) In Mental Maths:
   A) Children learn to select the best strategy.
   B) Children need to do exactly what the teacher tells them to do.
   C) There is no opportunity for discussion.

REFLECTION

• Think about the Mental Maths sessions in your class.
• How is mathematical language used during these sessions?
• How could you promote the learning of new strategies during these sessions?

Well done you have completed Lesson 6.
In this lesson, we will continue to discuss subtraction. Recognising the patterns of subtraction makes it easier for us to understand and solve problems. We will therefore look at different subtraction strategies and investigate using a number line to solve subtraction problems. Finally, we will consider how learner errors can help us to improve the teaching and learning happening in our classroom.

What you will learn in this lesson

- Subtraction strategies
- Subtraction using a number line
- Learner errors

Subtraction strategies

Initially, children will most likely be using counting strategies to solve problems. Let us look at the problem 8 - 3, and investigate the subtraction counting strategies that children could use:

- Counting back from: When using this strategy children count back from the first number. Children would count from 8..., 7, 6, 5
- Counting back to: Children would count back from 8..., 7, 6, 5, 4, 3 and count the numbers they have counted in order to arrive at the answer.
- Counting up from (complementary addition): Children begin by counting up from the smaller number 3..., 4, 5, 6, 7, 8

As we discussed earlier in Part 2, children need to learn to select the appropriate calculation strategies in order to solve problems efficiently. Teachers need to model these strategies and encourage children to move using counting as their only solution method.
Activity 1

Look at the subtraction strategies discussed above.

• Write down a subtraction story that you could use with your class.
• How could you help children to identify an appropriate strategy to use to solve the problem?
• What would you do if different children used different strategies to solve the problem?

Commentary

When giving children subtraction word problems, it is important to remember that there are different types of problems to which they should be exposed. Make sure you provide examples of all the different types of problems so that children can develop their understanding and confidence in all of them.

The first type is the combination type. In these problems, we know the total number of items, but we don’t know one of the parts. When solving these problems, part-part-whole diagrams are useful, and so is an understanding of inverse operations. For example, in a combine type problem, children may realise that the number sentence is $10 - 7 = \square$. They could then use the number sentence $7 + \square = 10$.

The second type of problem is the change type. In these types of subtraction problems, we generally have the situation where something leaves or breaks, leaving us with the question How many are left?. However, it is important that we provide a variety of problems where the unknowns are in different positions. For example, children may be asked to find out how many there were in the beginning, or to identify what the change was in the problem. Once again, a knowledge of inverse operations is useful here.

The third type of problem is the comparison type, and these are probably the most tricky for children to understand. In these questions, children are either asked to find out how many more or how many fewer there are. What children need to realise is that, regardless of the words ‘more’ or ‘fewer’, the question is actually wanting to know the amount between two numbers. As we discussed in Lesson 6, ‘difference’ problems are tricky for children, and number lines are helpful when solving them.

Subtraction using a number line

Learners need a conceptual understanding of subtraction – in other words, they need to know why they are solving a problem in a particular way. This can be achieved by learners explaining their actions, rather than solving problems by simply following a rote pattern or ‘recipe’.

Practicing subtraction on a number line helps children to understand that the numbers get smaller as we take quantities away. The number line provides a visual representation that allows them to recognise the patterns in their strategies. For example, in the problem $14 - 6$, children could count backwards for 6 jumps. They would then see where on the number line they landed. This is the simplest way they could try to solve the problem.

If the children have developed their understanding of number a bit more, they may choose to use ten as an anchor number. This means that they will jump 4 places to 10, and then another 2 places to 8.
ACTIVITY 2

Finding the Missing Number

Watch the video “Finding the Missing Number” (5:47 minutes) to see how you can use a number line to introduce new strategies.

- Describe an activity that you could do in your class that will help the children to learn how to find the missing number using a number line.

Commentary

As we have discussed previously, number lines are a useful way to help children to identify patterns. They can see the pattern of the numbers along the number line, as well as the patterns used in different ways of counting. In addition to this, children can also begin to notice the patterns in their solution methods. For example, in the video we see how jumps on the number line are used to help children solve addition and subtraction problems. They are used to this method, and therefore when they are asked to find a missing number using the number line, they already have a foundation of knowledge which can be used to help them.

The children would already be comfortable solving 6 + 4 on the number line. In the example from the video, they now have to find the missing number using a similar strategy. In doing this, they are reinforcing their understanding of number bonds, and the principles of addition and subtraction. By getting children to practice a variety of problems such as this, they are practicing the number facts shown below:

<table>
<thead>
<tr>
<th></th>
<th>6 + 4 = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 + 6 = 10</td>
</tr>
<tr>
<td></td>
<td>10 - 6 = 4</td>
</tr>
<tr>
<td></td>
<td>10 - 4 = 6</td>
</tr>
</tbody>
</table>

Learner Errors

It is important to understand the difference between mistakes and misconceptions in the classroom. Mistakes are incorrect answers due to errors in procedural workings. Learners make such mistakes from time to time, but these are easily corrected. Misconceptions arise from incorrect procedural or conceptual ideas, and they occur repeatedly. They are not easy to correct because learners believe that what they are doing is right when in fact, they are making errors. Teachers need to deal with learners’ misconceptions so that the learners will be able to progress to a more sophisticated mathematical understanding.
The diagram below illustrates the connection between ‘errors’, ‘mistakes’ (or slips) and ‘misconceptions’.

**ACTIVITY 3**

Look at the problem:
- What do you think the child's misconception might be?
- What could you do to rectify this misconception?

**Commentary**

In this example, the child has added the digits rather than subtracting them. In addition, he does not seem to understand the concept of place value. We can see that he has added 2 ones and 8 ones to get 10 ones. He also added 7 tens and 3 tens to get 10 tens. He then added 10 tens to 10 ones to get an answer of 1010.

He could use base ten blocks to help him visually recognise the value of the numbers, and to grasp the idea of equal exchange. By using these resources, the child will be able to physically handle the number and construct his own understanding. He could also use a number line to see the movement along the line, and to recognise that when you subtract, the numbers get smaller.
Check your understanding: True or False?

1) Counting strategies are the most effective ways to solve subtraction problems.  

2) The three types of subtraction problems include combine, change and comparison.

3) Number lines are useful visual representations of patterns.

4) When children make a mistake, it is always an indication of a serious misunderstanding of a concept.

REFLECTION

- Reflect on your experience of subtraction in the classroom.
- Do children typically find subtraction easy or difficult?
- What do you do when children make mistakes in their work?
- How can you tell if a child has made an error or if they have a misconception?

Well done you have completed Lesson 7.
In this lesson, we will continue discussing subtraction. We will look at the different concrete resources that can be used to help children construct their understanding of subtraction. We will then investigate the use of Flard cards in subtraction problems before talking about base ten blocks.

What you will learn in this lesson

- Subtraction and concrete resources
- Subtraction and Flard cards
- Subtraction and base ten blocks

Subtraction and concrete resources

In the same way as they did for addition, children can construct their understanding of subtraction by using concrete resources, before progressing on to pictorial representations. Once children have a sound understanding of the concept of regrouping, they can then move on to working symbolically. It is important that children do not move through the process too quickly, as this will lead to an increased chance of misconceptions being developed.

When children move through the process from concrete manipulatives to pictorial representations until they finally reach the abstract stage, they are able to build on their understanding at each step. This develops a firm foundation resulting in an ability to solve problems with greater confidence and efficiency. Children who do not have this foundation of understanding, tend to try and solve problems through rote learning, or memorisation of a ‘recipe’ or steps in solving a problem. The concern here is that, if children forget these steps, they then have no way to solve the problem. They cannot try a different method because they have no understanding on which to draw.

ACTIVITY 1

Describe an activity in which you can help children to develop their understanding of subtraction.

- Which concrete resources could you use in your activity to help children grasp the concept?
- How will the use of concrete resources help you assess the children?

Commentary

When selecting concrete resources for your activity, think about what you will be trying to teach. If you are working with smaller numbers, then you could use counters and ten frames. In this way children can physically experience the action of taking away counters, and they can see the resulting empty spaces on the ten frames.

If you will be teaching subtraction with larger numbers, then base ten blocks would be useful to represent the exchange process. We will discuss this further in Lesson 9. As the children use the resources, you as the teacher would observe what the children are doing. This will help you to assess the children's level of understanding. You will be able to see how they handle the resources, and this will give you a chance you to recognise their misconceptions. It is important to encourage children to verbalise what they are doing while they solve the problems. The expression of their thought process clearly indicates their ability to reason through a problem.
Subtraction with base ten blocks

When subtracting larger numbers, children can use base ten blocks with a place value mat as concrete manipulatives. Base ten blocks are a particularly useful resource for children to consolidate their understanding of regrouping. As we have mentioned, subtraction is tricky for children to grasp, and the idea of regrouping is challenging for them. They need to be able to see the process, and the be able to physically remove a ten from the tens column and exchange it for ten ones. Using the place value mat is helpful as they can then easily see where they need to go to in order to make the exchange.

In the images, we can see that the children realise that they only have 2 ones, but they need to take away 8. They know that they can exchange 1 ten for 10 ones, which then gives them 12 ones in total. They can then take 8 ones away, leaving them with 4 ones.

Equal exchange with money

In maths, there are a number of different topics, concepts and skills that are covered. Very often, teachers approach these separately, teaching each one on its own, before moving on to the next idea. Unfortunately, this approach doesn’t allow children to see the inter-related nature of these topics, skills and concepts. As previously mentioned, the more children know about one number, the more they understand about that number’s relationship to other numbers. In the same way, a topic such as Money should not be taught in isolation. If the concept of Money is only taught for a few lessons before the teacher moves on to a different topic, then the children are likely to forget what they have learnt. It is a good idea to use what children know about notes and coins to help them develop their understanding of number, as seen in our Smart Counting lessons. Children can use R10 notes to help them count in tens or they can use R2 coins to help them count in twos. These examples will help children see the R10 note as a group of ten, recognising that even though it is only one note, it has the same value as ten R1 coins. This will encourage children to progress from their initial stage of counting all.

ACTIVITY 2

Subtraction Part 1 - using base ten-blocks (no exchange)

Watch the video “Subtraction Part 1 - using base ten-blocks (no exchange)” (4:13 minutes) to see how base ten blocks can be used to solve subtraction problems.

- Have you used, or seen children using, either of the two methods shown in the beginning of the video?
- Why do you think children find the concept of subtraction difficult?
Commentary

The two strategies shown at the beginning of the video might not be strategies that you have seen children use in the classroom, but it is important for you to know and understand them. As children develop their understanding of subtraction, you may find that one of these methods is used to solve a problem. This is why it is so important to allow children the opportunity to verbalise their methods, so that you can determine what level of understanding they have, and also for other children to hear different methods.

Subtraction is a tricky concept from children, because typically children rely on their knowledge of addition in order to develop their understanding of subtraction. This means that children need a solid foundation of solving a variety of addition problems so that they can confidently begin building on their ideas about subtraction. Unfortunately, this can also mean that some children may make errors in their attempts to solve problems by using learnt addition strategies rather than selecting a method based on their understanding of the problem.

Subtraction and Flard cards

Flard cards are very useful resource in the classroom. The cards help children to recognise the value of each digit in a number, and it also helps them to break down numbers in such a way that it simplifies the calculation process.

When children recognise that 499 is broken down into 400 + 90 + 9, it becomes far easier for them to subtract 10. They just need to take ten away from the ninety, leaving them with 80. Then they can reconstruct the number, saying 400 + 80 + 9 = 489.

Flard cards help children to make the transition from base ten blocks to the abstract written number sentences and column format. It is therefore important that children have a sound understanding of subtraction with base ten blocks, counting in multiples and regrouping before they start trying to move towards more abstract representations.

ACTIVITY 3

Describe how you would use Flard cards with the children in your class.

• How would you get children to prepare for the start of the activity?
• How can you get children to consolidate their understanding of subtraction through a game?

Commentary

It is a good idea to have a large set of Flard cards for whole class demonstrations, so that children can visually follow the discussions. Children can then use their own smaller versions of the Flard cards so that they can be actively involved in the activity. Children should sort the cards into groups according to their place value. The children can lay the cards out in order so that when they are looking for a particular card, it will be easier to find.

You can consolidate children's understanding of place value by calling out numbers and getting children to construct the number with their Flard cards. You need to ask children to break down numbers (321 into 300 and 20 and 1) as well as build up numbers (200 and 10 and 3 makes 213). You could also ask the children some quick mental calculations such as Show me 10 more than 431 and Show me 5 less than 675.

Children can also play a game where they work in pairs, and they each have to create a number using their Flard cards. Once their numbers have been constructed, they can then add or subtract their numbers. You can get the children to keep a tally record of who solves each problem the fastest.
## Check your understanding: Multiple Choice

1) **Accurate and efficient subtraction involves:**
   - A) Rote learning.
   - B) Following a recipe.
   - C) Building up understanding progressively.

2) **Base ten blocks:**
   - A) Are useful to see the process of exchange.
   - B) Too bulky and awkward to use in a classroom.
   - C) Are only useful for addition problems.

3) **Children find subtraction:**
   - A) Easier than addition.
   - B) Difficult because subtraction is not logical.
   - C) Difficult because children need to rely on their understanding of addition.

4) **Flard cards:**
   - A) Develop children's understanding of place value.
   - B) Should not be used in subtraction problems.
   - C) Both A and B.

---

## REFLECTION

- Think about your use of concrete resources when teaching subtraction.
- Which resources do you prefer using in the classroom?
- Which resources do the children prefer using to help them?
- What can you do to manage your use of resources more effectively?

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**Well done you have completed Lesson 8.**
In this lesson, we will revisit the idea of pictorial representation. Children will move towards using pictures to solve problems, rather than concrete apparatus, as they consolidate their understanding. It is therefore important for us to have prepared for this progression of learning. We will also discuss using games to help children learn, as well as discussing the role of continuous assessment in Foundation Phase classrooms.

**What you will learn in this lesson**
- Subtraction and pictures
- Games
- Assessment

**Subtraction and pictures**

As we discussed with addition, children will eventually begin to move away from concrete resources and begin to use pictorial representations instead. As they consolidate their understanding and become more confident in their ability to solve problems, they will find it easier to draw pictures to solve problems.

Drawing base ten blocks is a more abstract way of solving problems, and children will find it quicker to draw squares, lines and dots to represent hundreds, tens and ones.

Children can use a 100 square to subtract numbers. They will be able to see the patterns that arise from subtracting numbers, and they will learn to use these to help them solve problems (first picture below). Children can also use a number line to solve subtraction problems. They can develop efficient strategies by using the number line, and these will help them to solve problems quickly and accurately (second picture below).
ACTIVITY 1

Subtraction Part 3 - using base-ten blocks (with exchange)

Watch the video “Subtraction Part 3 - using base-ten blocks (with exchange)” (7:20 minutes) to see how children can solve a subtraction problem that involves equal exchange with their base ten blocks.

• What do you notice the children do in the beginning of the video?
• How do children solve the problem 72 - 45?

Commentary

In the beginning of the video, the children initially pack out 72 and 45. The teacher needs to stop the children and remind them that in addition we pack out both numbers, but that in subtraction we only pack out the first number. As we discussed in our previous lesson, this shows us that children rely on their knowledge of addition in order to solve subtraction problems. It is essential that the children are given plenty of time to develop their understanding of the concept of subtraction, so that they are able to choose the appropriate strategy to solve the problem.

In the video, we can see that children practice equal exchange in order to solve the problem. There seem to be few children who are confident in this, and so the teacher models the process for the whole class. It is important to note that children may forget to subtract the tens one they have exchanged and subtracted the ones. Many children tend to think they have then finished the problem, forgetting to subtract the tens. When teaching this strategy, be sure to reinforce this and to look out for children who may develop misconceptions.

Games

As we have discussed throughout our lessons, children learn best through play. By playing games, and engaging with resources, children are able to construct their own understanding of concepts. Most activities can be turned into a game, where children can solve problems in a fun way. When children play, they learn without realising it. They are able to develop their understanding without trying to memorise strategies. They use their knowledge of number and concepts to find ways to get to answers, which helps them to recognise strategies for themselves.

ACTIVITY 2

How could you provide opportunities for children to learn their bonds of 10 in a playful manner?

• Describe a game that children could play using number and picture cards from 0 – 10.
Commentary

As we have learnt in Part 1, children learn through play. Children need a variety of opportunities to practise using their mathematical vocabulary in playful situations. The teaching and learning of number bonds is a great way for children to develop their ability to verbalise their understanding, and to participate in game playing. When learning their bonds of ten, children could be given a set of numeral cards and picture cards from zero to ten. Children could work in pairs, turning their cards face down on the table. They could then play a memory game where they try to match cards to make the sum of ten. As they play, they’ll be able to verbalise their understanding of the relationship between numerals and pictures, using the correct mathematical vocabulary to express their thinking.

Assessment

The simple answer to the question, ‘When should assessment take place?’ is that assessment should be ongoing and continuous. Assessment is central to the teaching and learning cycle, but the purpose of the assessment at different points in teaching and learning will be different.

Before beginning teaching on a new topic, you need to determine what the learners already know about what is to be taught. It is also good to give them some insight into how they will benefit from what they are about to learn. Secondly you need to determine the prior knowledge that the learner has already acquired in relation to the topic.

During the lesson, one should do ‘in-process assessment’ (also known as ‘feedback’ or ‘formative assessment’). This is important as it provides information on the learner’s progress on an ongoing basis. It also indicates to teachers and learners if the concepts and skills being taught have or have not been learned and is used in order to plan follow-up teaching and learning.

During this phase, assessment is undertaken at specified times after teaching and learning have taken place. The learners’ achievements are then communicated to them, their parents and the school personnel. This type of assessment can be classified as summative assessment since it provides information about the learning achievements in relation to the curriculum requirements. Sources of obtaining summative assessment results include things such as oral or written tests, and records of structured oral and practical activities. The goal of the assessment process is to support all learners to attain the important mathematical knowledge for the term and grade as it progresses sequentially along the learning trajectories designed into the curriculum.

ACTIVITY 3

- Why do you think that assessment in the Foundation should take the form of continuous assessment?
- Can continuous assessment be used as summative assessment?

Commentary

Continuous assessment is the natural form of assessment in Foundation Phase classrooms, because the children are at the beginning of a journey of lifelong learning. As they journey, they will learn the values, attitudes, skills and knowledge that they need to achieve the goals they set for themselves in life. It is the responsibility of teachers to ensure that they do indeed achieve this learning, as they progress through school.
Formative assessment is a part of continuous assessment – it involves feedback, which can be given if assessment is carried out on a continuous basis, using a variety of methods. Children do not all learn things at the same rate. Factors such as the situation they find themselves in and their individual ability come into play. Children should be given several opportunities to show that they are progressing in the achievement of the learning outcomes. A policy of continuous assessment facilitates the formative use of assessment.

To rely on a final test at the end of a term or year will not allow all children to demonstrate the range of knowledge and skills they have developed, and the stages of their progression. Continuous assessment gives you the opportunity to vary the kind of assessment you are using because you assess the children a number of times and in different ways. The results of all these can be used in the final (summative) assessment of children’s achievement.

Check your understanding: True or False?

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1) Number lines, drawings and 100 squares are good examples of pictorial representations that can be used when teaching subtraction.</td>
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<tr>
<td>2) In Maths, games should be limited to specific days, times or topics.</td>
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<tr>
<td>3) Assessment should only take place after a topic or concept has been taught.</td>
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<tr>
<td>4) Summative assessment is the only type of assessment that provides accurate information.</td>
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</table>

REFLECTION

- Think about the way you assess learners in your class.
- Do you follow the guidelines suggested above for assessing before, during and after teaching and learning?
- Discuss at least one idea that you learned about when doing the activities in this session that you can apply in your teaching.

Well done you have completed Lesson 9.
In this lesson, we will spend time looking at solving addition and subtraction problems by using the number line. We will expand on our previous discussions of this and consider how we can use the number line to solve problems more efficiently. We will then investigate using inverse operations to check our solutions to addition and subtraction problems. Finally, we will think about how learner errors help us to plan maths lessons in order to address misconceptions.

What you will learn in this lesson
• Addition and subtraction using the number line
• Using inverse operations to check
• Learner errors

Addition and subtraction using the number line

When children try to subtract numbers in an abstract way, they can make mistakes due to misconceptions or simple errors. If they use a number line, they can begin to think of efficient ways to solve the problem that result in fewer errors.

For example, in the problem 723 – 396, the children need to exchange both ones and tens. This is a skill which can be tricky for children, and so can result in errors. However, if the children use a number line, they can partition numbers in such a way that makes subtracting and bridging the ten easier to manage. Child may decide to count backwards along the line, by counting back in tens first. They could then take away 300. To bridge the ten, children may then break the 6 into 3 and 3.

Some children may find it more efficient to use flexible partitioning, where they break 396 into chunks that make it easier for them to move along the number line. Some children may decide to break 396 into 300 + 70 + 20 + 3 + 3 so that they could bridge the tens in a way that makes sense for them.
ACTIVITY 1

Addition and Subtraction using the Number Line Grade 3

Watch the video “Addition and Subtraction using the Number Line Grade 3” (5:44 minutes) to see how children can use a number line to solve addition and subtraction problems.

- What do you notice about the number line used in the video?
- Why is it important to allow the children to select the intervals for the number line?

Commentary

In this video, the children use an open number line. This means that, while there are interval demarcations, there are no numbers indicated on the line. This is so that children can fill in their own numbers, and they can decide on the intervals. This is a useful resource, as it gets children to think about counting in multiples, and about choosing the best strategy to solve a problem. If you had the problem 53 + 30, and you decided to go up in ones for your number line intervals, you would need a very long number line to fit everything on. If you use tens as your intervals, then it makes the number line much more practical.

![Hundred Square]

It is necessary to allow children opportunities to count in multiples, where they start from a different number. For example, if you wanted children to count in tens, it is a good idea to get them to start at 7 so they need to say 7, 17, 27, 37, 47, 57 etc. Using a hundred square is a useful resource for this, so that children can see the pattern as they move down the column of numbers.

Using inverse operations to check

Inverse means the opposite, and a mathematical operation is the way in which we calculate. The four operations are addition, subtraction, multiplication and division. So, when we refer to an inverse operation in maths, we are talking about an operation that will undo what was done by the previous operation. For example, if we have 5 pencils and we add another 2 pencils, we end up with 7 pencils all together. The inverse of this would be to take 7 pencils and subtract 2 pencils. We would be left with 5 pencils. In this example, addition and subtraction are inverse operations.
Addition and subtraction are inverse operations, so addition ‘undoes’ subtraction and subtraction ‘undoes’ addition. This means that we can use addition and subtraction to check our answers when we’re solving problems. Children can rearrange the numbers in a subtraction number sentence to create two addition number sentences, which can then be used as part of their check after they have solved a problem. For example, if children are given the problem $45 - 34$, children could use base ten blocks or a number line to help them get to an answer of 23. They would write out the number sentence:

$$45 - 34 = 11$$

Children could then rearrange the numbers into:

$$11 + 34 = 45$$ and $$34 + 11 = 45$$

The addition problem could be solved by a quick mental addition of $34 + 10 + 1 = 45$, which tells the children that they have solved the subtraction problem correctly.

**ACTIVITY 2**

Give children a subtraction problem to solve.

- Watch children solve the problem and see their level of understanding.
- Ask children to use addition to check their solution and see whether they are able to use the inverse operation to clarify their understanding.

**Commentary**

The operations have certain rules (or ‘laws’) that you need to know about. The children do not need to know the formal names of these laws, but they do need to be aware of them from quite an early stage. It is important that you as the teacher know the names and the functioning of these laws.

Addition and subtraction are inverse operations, but they do not behave in exactly the same way. Addition is commutative which means that we can add a pair of numbers in any order and still get the same answer. However, subtraction is not commutative. Addition is associative which means that when we add three or more numbers together, we can pair them in any order we choose, without changing the final answer. Subtraction is not associative.

We can use addition and subtraction to check our answers to problems. A part-part-whole diagram is a visual representation of the notion of using inverse operations to check solutions to problems.

<table>
<thead>
<tr>
<th></th>
<th>37</th>
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<tbody>
<tr>
<td>25</td>
<td>12</td>
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</table>

Children can see that: $25 + 12 = 37$  But they can also see that: $37 - 25 = 12$

$12 + 25 = 37$  $37 - 12 = 25$
Learner Errors

According to Ball there are different types of knowledge involved in doing error analysis. Subject knowledge (which Ball and Shulman call content knowledge) enables the teacher to identify an error (is the answer right/wrong?). Identification of an isolated error is just the first step. If a teacher can identify patterns in learners’ work, this shows a deeper understanding of the content from the perspective of the learner. This is what is known as specialised content knowledge. If the teacher can realise what learners were thinking when they made the error, then the teacher is using pedagogic content knowledge. This is a kind of knowledge which is special to teachers since it enables them to understand their learners more deeply and be able to communicate with them more effectively.

The implication of this is that teachers need to explore and diagnose misconceptions. They need to understand where misconceptions and errors come from, and then plan mathematics lessons that will confront children’s misconceptions. If teachers can get children to engage in a dialogue and restructure their thinking, then they are on the correct path to correcting misconceptions. Teachers also need to allow children to make mathematical connections between any new knowledge and their own meanings that they have constructed. In order to do this, it is necessary to allow children to compare their conceptions to those of other children. This will help them to broaden and consolidate their understanding of the concept.

ACTIVITY 3

Give your children a subtraction problem to solve. For example:

Thandeka has 17 chickens.
Lindiwe has 9 chickens.

What is the difference in the number of chickens?

• Observe how the children solve the problem.
• Can you identify what knowledge the children need to have in order to solve this problem accurately?

Commentary

In this problem, the children have to recognise what the question is actually asking them to do. They need to know the mathematical vocabulary (‘difference’), recognising that they are being asked to find out how many numbers are in between 9 and 17. In order to solve this problem, children need to be able to select the appropriate resources to help them. A number line is a useful resource as it clearly shows the children how many numbers are in between 9 and 17. Children also need to have an understanding of inverse operations, because they could solve this problem by saying $17 - 9 = \square$, or they could say $9 + \square = 17$.

As the teacher, if you noticed your children adding 9 and 17, you would need to then decide what that tells you about their understanding of addition and subtraction. The children may have a limited understanding of mathematical language, which you could focus on in future lessons to correct. Alternatively, children may have a misconception about how to solve subtraction problems. They may be trying to use addition strategies to solve all problems, regardless of the nature of the question, because they have not yet grasped the concept of subtraction. This requires a different direction in future lessons in order for you to address the misconception.
Check your understanding: Multiple Choice

<table>
<thead>
<tr>
<th>1) When using a number line:</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>A) Children should count in ones.</td>
<td>B) Children need partition numbers into hundreds, tens and ones only.</td>
<td>C) Children can choose to flexibly partition numbers.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2) An open number line:</th>
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<th></th>
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</table>

<table>
<thead>
<tr>
<th>3) Inverse operations:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Mean undoing what was done.</td>
<td>B) Can be used to check solutions.</td>
<td>C) Both A and B.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4) Children’s incorrect answers:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Help the teacher determine the direction of future lessons.</td>
<td>B) Should be ignored as the children are just careless.</td>
<td>C) Are only important if most children in the class make the same mistake.</td>
</tr>
</tbody>
</table>

REFLECTION

- Take some time to think about your own practices in relation to learners who make errors in your mathematics class.
- Reflect on your teaching practices with these questions in mind:
  - How do you address learners’ errors?
  - Could you engage with learners’ errors more effectively? Why or why not?

Well done you have completed Lesson 10.